Realistic shell-model calculations for neutron-rich calcium isotopes

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Why to perform realistic shell-model calculations for calcium isotopes?

- To ascertain if modern realistic shell-model potentials are able to overcome the deficiences of the previous ones
- If the answer is in the affirmative, to investigate the shell-structure of these nuclei lying far from the stability valley

L. Coraggio, A. Covello, A. Gargano, and N. Itaco, Phys. Rev. C **80**, *044311 (2009)*

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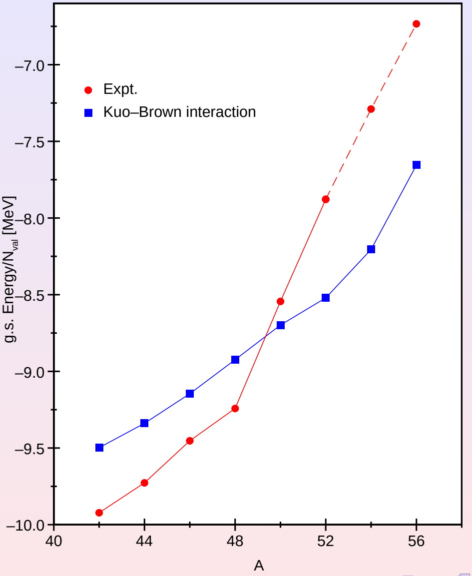
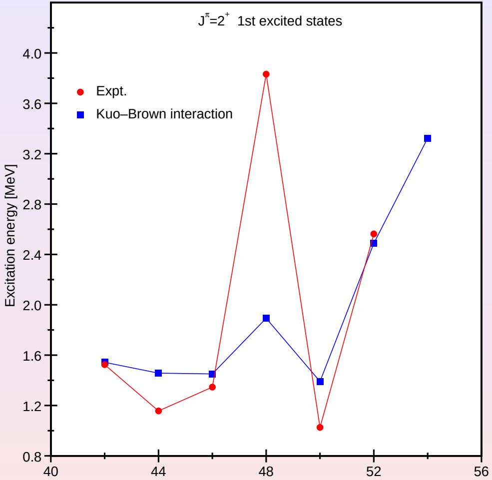


Expt.
Kuo–Brown interaction
g.s. Energy/N$_{val}$ [MeV]
A
−7.0
−7.5
−8.0
−8.5
−9.0
−9.5
−10.0
40
44
48
52
56


$J^{\pi}=2^{+}$ 1st excited states
Expt.
Kuo–Brown interaction
Excitation energy [MeV]
4.0
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
40
44
48
52
56

There has been a significant progress about the derivation of realistic effective shell-model hamiltonians:

- Renormalization of modern *NN* potentials by way of the $V_{\rm low-k}$ approach
- A sound theory to derive the effective shell-model hamiltonian: $\hat{Q}$-box plus folded-diagram method

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Low-momentum nucleon-nucleon potentials: the $V_{\mathrm{low-k}}$

Inspiration to renormalize V_{NN}:

- Effective field theory (EFT)
- Renormalization group (RG)

from EFT: we restrict the configurations of $V_{NN}(k, k')$ to those with $k, k' < k_{\mathrm{cutoff}} = \Lambda$

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The two-nucleon hamiltonian:

In the full space:

$$\int_0^\infty [H_0(k,k') + V_{NN}(k,k')] \langle k|\Psi_\nu\rangle k^2 dk = E_\nu \langle k'|\Psi_\nu\rangle$$

In a reduced model space $P = \int_0^\Lambda |k\rangle\langle k| k^2 dk$

$$\int_0^\Lambda [H_0(k,k') + V_{\rm low-k}(k,k')] \langle k|\Phi_\mu\rangle k^2 dk = \tilde{E}_\mu \langle k'|\Phi_\mu\rangle$$

Fundamental constraint: $\tilde{E}_\mu \in \{E_\nu\}$

How to construct $\langle k|H_{\rm eff}|k'\rangle$?

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Unitary transformation: Andreozzi-Lee-Suzuki method

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A numerical test: phase shifts in the $^1S_0$ channel (in degrees)

E_{lab} (MeV)	CD-Bonn	$V_{\mathrm{low-k}}$	Expt.
1	62.1	62.1	62.1
10	60.0	60.0	60.0
25	50.9	50.9	50.9
50	40.5	40.5	40.5
100	26.4	26.4	26.8
150	16.3	16.3	16.9
200	8.3	8.3	8.9
250	1.6	1.6	2.0
300	-4.3	-4.3	-4.5

The shell-model effective hamiltonian

A very useful way to derive H_{eff} is the time-dependent perturbative approach as developed by Kuo and is co-workers in the 1970s (see *T. T. S. Kuo and E. Osnes, Lecture Notes in Physics vol. 364 (1990)*)

In this approach the effective hamiltonian H_{eff} is expressed as

$$H_{\text{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots ,$$

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- The integral sign represents a generalized folding operation

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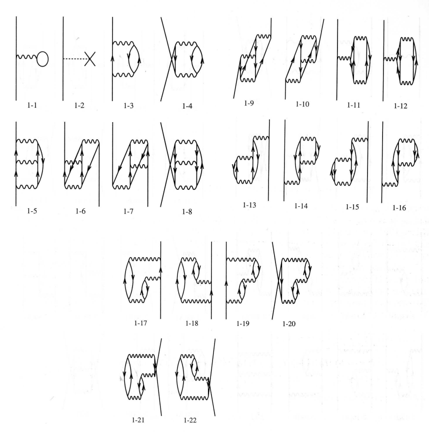
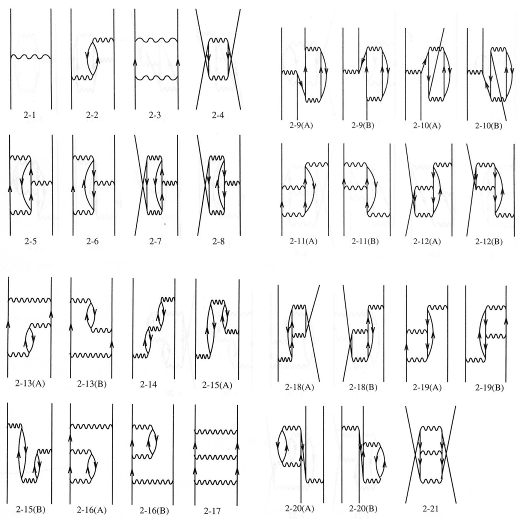


M. Hjorth-Jensen, E. Osnes, and T. T. S. Kuo, Phys. Rep. **261** *(1995)*


2-1
2-2
2-3
2-4
2-9(A)
2-9(B)
2-10(A)
2-10(B)
2-5
2-6
2-7
2-8
2-11(A)
2-11(B)
2-12(A)
2-12(B)
2-13(A)
2-13(B)
2-14
2-15(A)
2-18(A)
2-18(B)
2-19(A)
2-19(B)
2-15(B)
2-16(A)
2-16(B)
2-17
2-20(A)
2-20(B)
2-21

Shell-model hamiltonian for calcium isotopes

- Input V_{NN}: $V_{\mathrm{low-k}}$ derived from the high-precision *NN* CD-Bonn potential.
- H_{eff} obtained including diagrams up to the 3rd order in $V_{\mathrm{low-k}}$.
- Effective electric quadrupole operator calculated at 3rd order in perturbation theory, consistently with H_{eff}.
- Single-particle spacings fixed to reproduce the observed energies of single-particle states in ^{47}Ca and ^{49}Ca; absolute energies fixed so to reproduce experimental binding energy of ^{49}Ca with respect ^{48}Ca.

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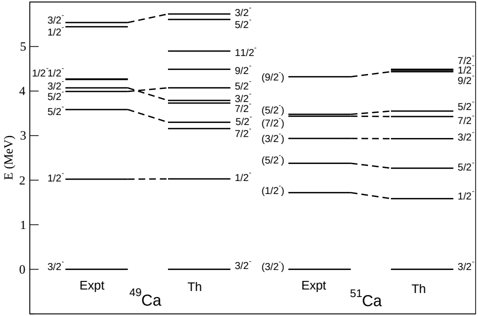
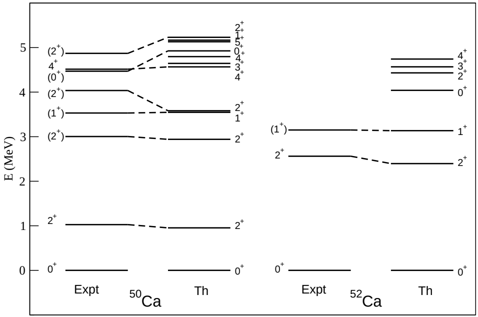
Single-particle energies

Orbital	SP energies (in MeV)
$0f_{7/2}$	0.0 (-8.2)
$1p_{3/2}$	2.7
$1p_{1/2}$	5.5
$0f_{5/2}$	8.5

Odd-mass neutron-rich isotopes

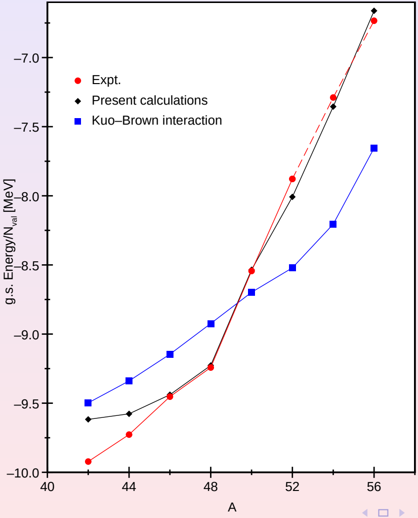


Even-mass neutron-rich isotopes

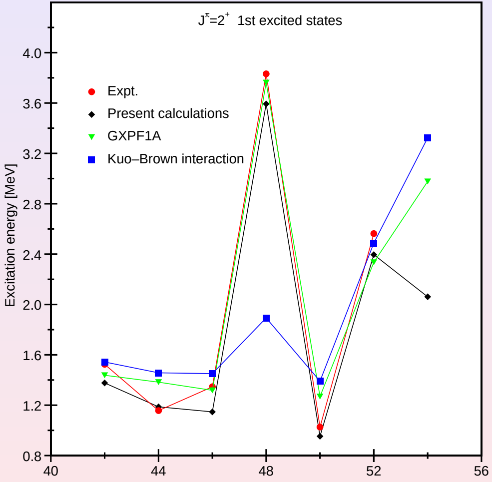


In ^{50}Ca: $B(E2; 2_1^+ \rightarrow 0_1^+) = 10.9\ e^2\mathrm{fm}^4\ (7.5 \pm 0.2\ e^2\mathrm{fm}^4)$

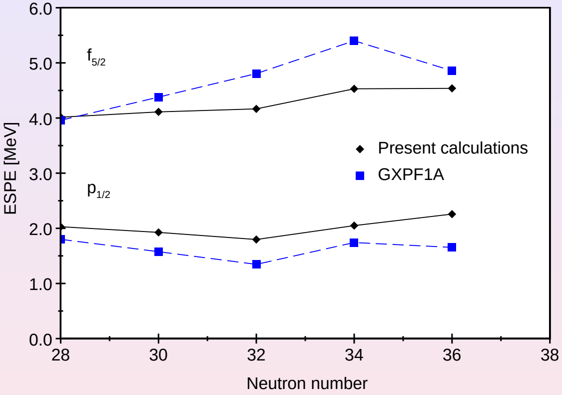
Saturation properties: ground-state energies per valence neutron



Shell-closure properties: excitation energies of $J^\pi = 2_1^+$



Effective single-particle energies



$f_{5/2}p_{3/2}$=-0.09 MeV (present calculations)

$f_{5/2}p_{3/2}$=0.12 MeV (GXPF1A)

Concluding remarks

- The agreement of our reults with the experimental data testifies the reliability of our $V_{\rm eff}$
- No need of $T=1$ monopole corrections
- No evidence of the magic number $N=34$
- Looking forward for more experimental data for neutron-rich calcium isotopes

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Calculations for valence proton-neutron nuclei: Ti isotopes