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IMPROVED METHOD OF ANALYSIS FOR RECOIL–DISTANCE MEASUREMENTS OF NUCLEAR LIFETIMES

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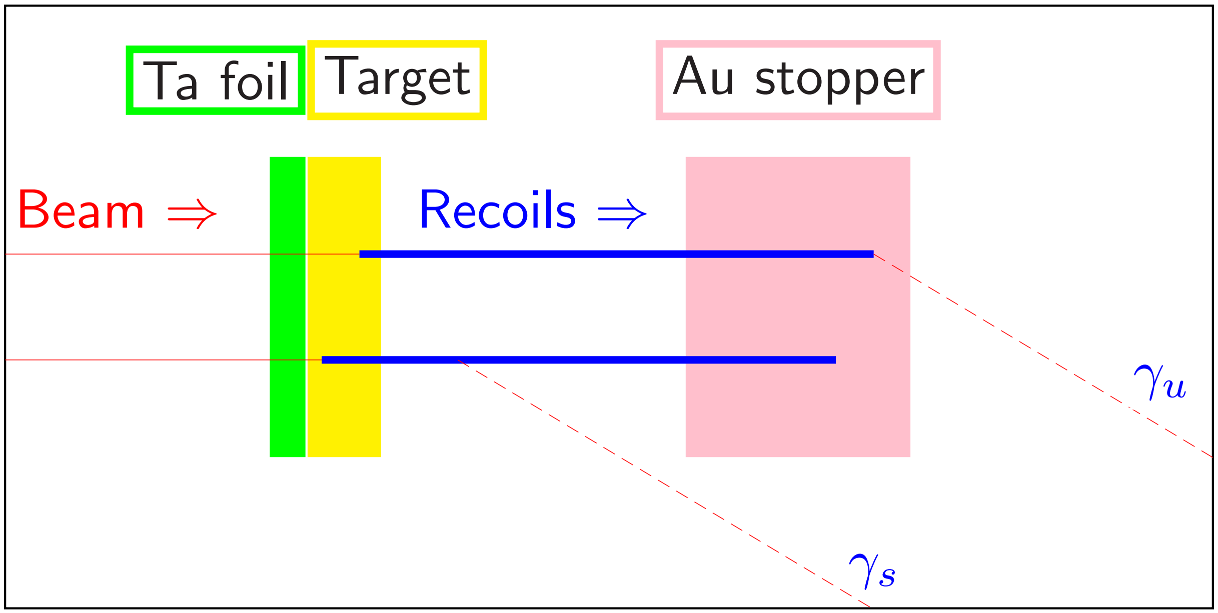
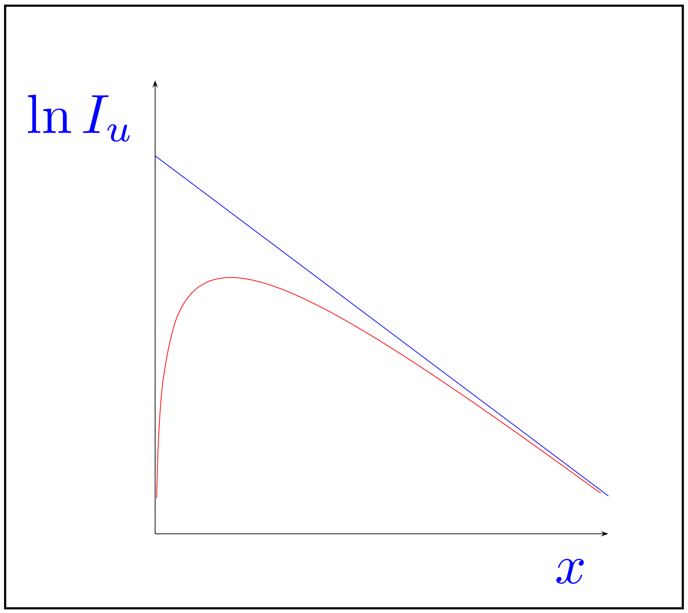
The Recoil Distance Method





If only one relevant lifetime:

$I_u = I_0 \exp[-x/(v\tau)]$

But: Effect of delayed feeding

$$E(\gamma_s) = E(\gamma_u)\ [1 + (v/c)\ \cos\theta]$$

Data analysis (Differential Decay Curve Method [1])

$$I_u^{(s)}(t) = \int_0^t f(t') \exp\left(\frac{t-t'}{\tau}\right) \mathrm{d}t'$$

$$I_s^{(s)}(t) = \int_0^t f(t') \left[1 - \exp\left(\frac{t-t'}{\tau}\right)\right] \mathrm{d}t'$$

$$I_u^{(s)}(t) = \boldsymbol{\tau} \frac{\mathrm{d}I_s^{(s)}(t)}{\mathrm{d}t} = v\boldsymbol{\tau} \frac{\mathrm{d}I_s^{(s)}(x)}{\mathrm{d}x}$$

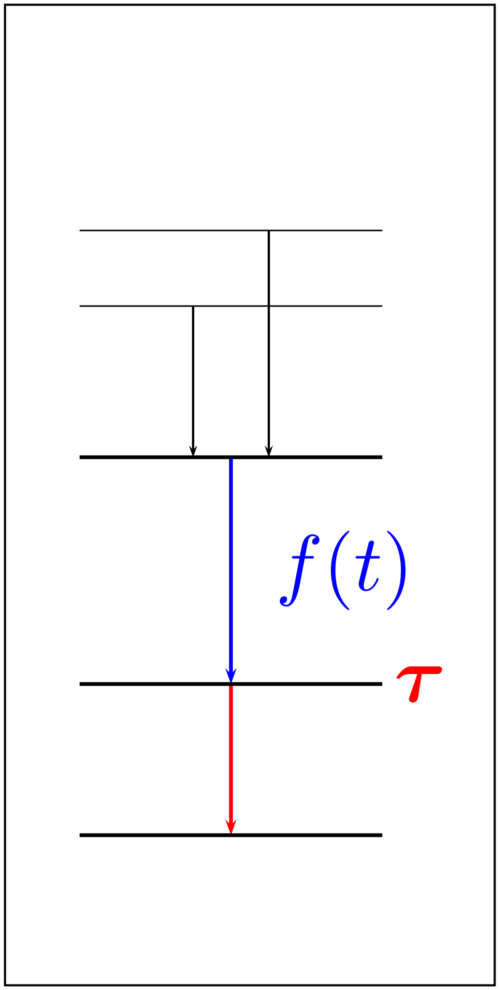
Where

$\boldsymbol{t = x/v} \Rightarrow$ time of flight (v assumed to be constant).

$\boldsymbol{f(t)} \Rightarrow$ Decay curve of the feeding transition,

$\boldsymbol{I_s^{(s)}(t)} \Rightarrow$ Nr of γ rays emitted in flight,

$\boldsymbol{I_u^{(s)}(t)} \Rightarrow$ Nr of γ rays emitted in the stopper (including those emitted during the slowing-down) in coincidence with in-flight part of feeding transition.

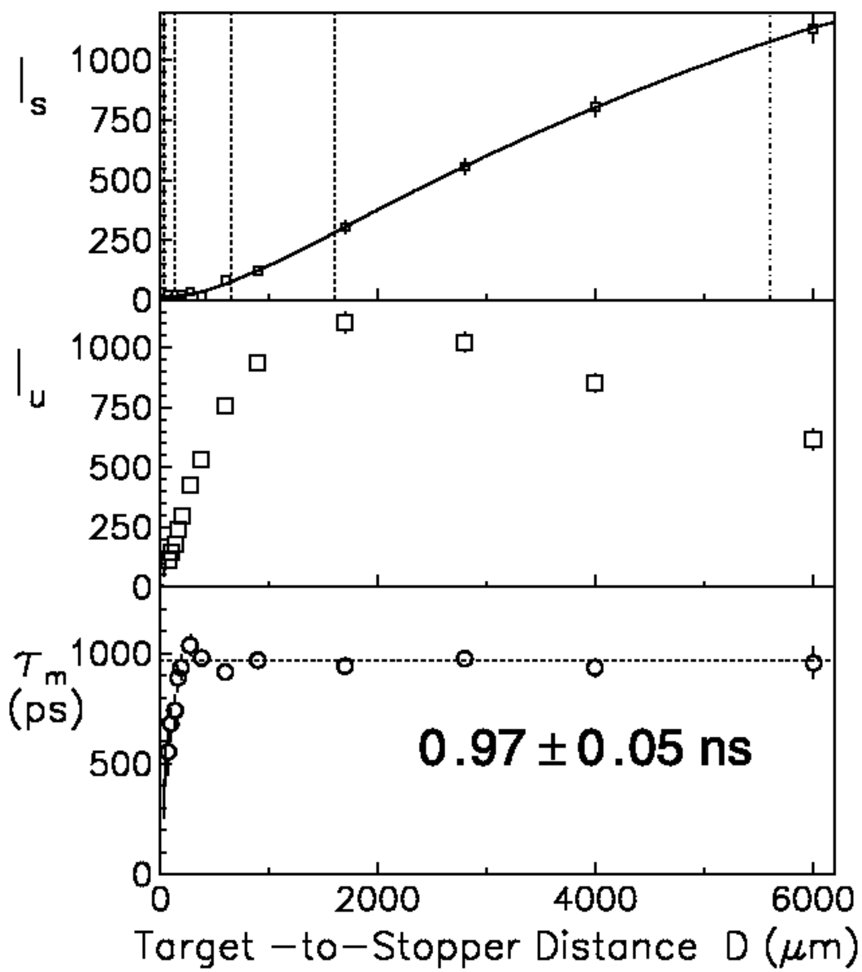


[1] A. Dewald *et al.*, Z. Phys. A **334**, 163 (1989);

G. Böhm *et al.*, N.I.M. A **431**, 208 (1999).

A typical example of DDCM analysis [2]

^{146}Gd 9^-



[2] Nuovo cimento A **111, 697 (1998)**

The curve of shifted intensity $I(D)$ is fitted with a quadratic spline (nodes at vertical dashed lines).

Each experimental point gives a value of $\tau = I_u(D)/[\mathrm{d}I_s/\mathrm{d}D]$ (with derivatives from spline).

Problems:

* The (average) velocity is assumed to be independent of D.
* Uncertainties in derivatives can be large at the two borders of the significant region.

With very small beam current (as with radioactive beams):

– Only one (or few) τ can be measured.
– The (very pleasant!) redundancy of DDCM cannot be achieved:
preferable one single result with sufficient statistics!
– Thicker targets $\Rightarrow$ Large spread in the velocity of recoiling nuclei.

In the place of $I_u(x) = v\tau \, \mathrm{d}I_s(x)/\mathrm{d}x$ we can use the integral relation:

$$\int_{x_1}^{x_2} I_u(x)\mathrm{d}x \quad = \quad \boldsymbol{\tau} \left[\bar{v}_x(x_2)\ I_s(x_2) - \bar{v}_x(x_1) I_s(x_1)\right]$$

In principle, one could use also the differential form

$$I_u(x) \quad = \quad \boldsymbol{\tau}\, \frac{\mathrm{d}\left[\bar{v}_x(x)\ I_s(x)\right]}{\mathrm{d}x}$$

and follow the usual DDCM procedure.
Both relations hold also in the presence of a broad distribution of velocities, if $\bar{v}_x(x)$ is the *average* velocity of nuclei decaying in flight for a distance x.
The average velocity $\bar{v}_x(x)$ can be deduced from the observed Doppler shift of γ transitions in the coincidence spectra used to evaluate I_s and I_u.

In fact: assume $g(x, v_x)\mathrm{d}v_x\mathrm{d}x$ gives the number of nuclei with velocity component (along the beam direction) in the interval $v_x, v_x{+}\mathrm{d}v_x$, whose decay feeds the relevant level while they lie in the space interval $x, x{+}\mathrm{d}x$:

$$\begin{aligned} I_u(x) &= \int_0^x \mathrm{d}x' \int_{v_0}^{\infty} \mathrm{d}v_x\; g(x', v_x) \exp\left(-\frac{x-x'}{v_x \tau}\right) \\ I_s(x) &= \int_0^x \mathrm{d}x' \int_{v_0}^{\infty} \mathrm{d}v_x\; g(x', v_x) \left\{1 - \exp\left(-\frac{x-x'}{v_x \tau}\right)\right\} \end{aligned}$$

We integrate I_u from 0 to X_0:

$$\begin{aligned} \int_0^{X_0} I_u(x)\mathrm{d}x &= \int_0^{X_0} \mathrm{d}x' \int_{v_0}^{\infty} \mathrm{d}v_x g(x', v_x) \int_{x'}^{X_0} \mathrm{d}x\;\; \exp\left(-\frac{x-x'}{v_x \tau}\right) \\ &= \int_0^{x} \mathrm{d}x' \int_{v_0}^{\infty} \mathrm{d}v_x\; g(x', v_)\; v_x \boldsymbol{\tau} \;\left[1 - \exp\left(-\frac{X_0-x'}{v_x \tau}\right)\right] \\ &= \boldsymbol{\tau}\; I_s(X_0)\; \bar{v}_x(X_0) \\ \text{where } \bar{v}_x(X_0) &= \frac{\int_0^{X_0} \mathrm{d}x' \int_{v_0}^{\infty} \mathrm{d}v_x\; v_x\; g(x', v_x) \left[1 - \exp(-\frac{x-x'}{v_x\tau})\right]}{\int_0^{X_0} \mathrm{d}x' \int_{v_0}^{\infty} \mathrm{d}v_x\; g(x', v_x) \left[1 - \exp(-\frac{x-x'}{v_x\tau})\right]} \end{aligned}$$

The average velocity component $\bar{v}_x(X_0)$ is just the one that can be deduced from the average Doppler shift at the distance X_0. The difference of the integrals for $X_0 = x_2$ and $X_0 = x_1$ gives $\int_{x_1}^{x_2} I_u(x)\mathrm{d}x = \boldsymbol{\tau}\left[I_s(x_2)\bar{v}_x(x_2) - I_s(x_1)\bar{v}_x(x_1)\right]$

Finally, deriving both terms of this equation with respect to x_2 one obtains

$$I_u(x) = \boldsymbol{\tau} \frac{\mathrm{d}}{\mathrm{d}x} \left[\; I_s(x)\; \bar{v}_x(x)\right]$$

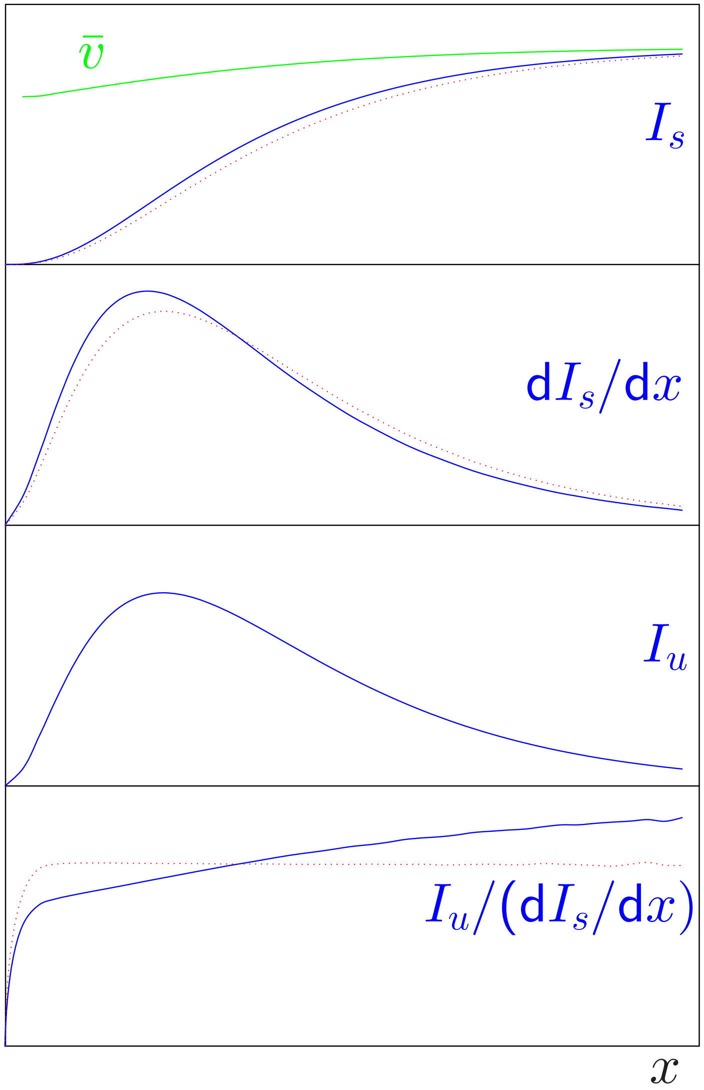
which is equivalent to the standard form of DDCM only if $\bar{v}_x$ is independent of x.

How large can the correction be?

The calculated curves show the effect of a wide spread in v_x (from $0.5v_0$ to $1.5v_0$).

Existing Monte-Carlo codes [1] can provide a best fit of the ensemble of results, with good statistical efficiency but with systematic uncertainties in the slowing down process.

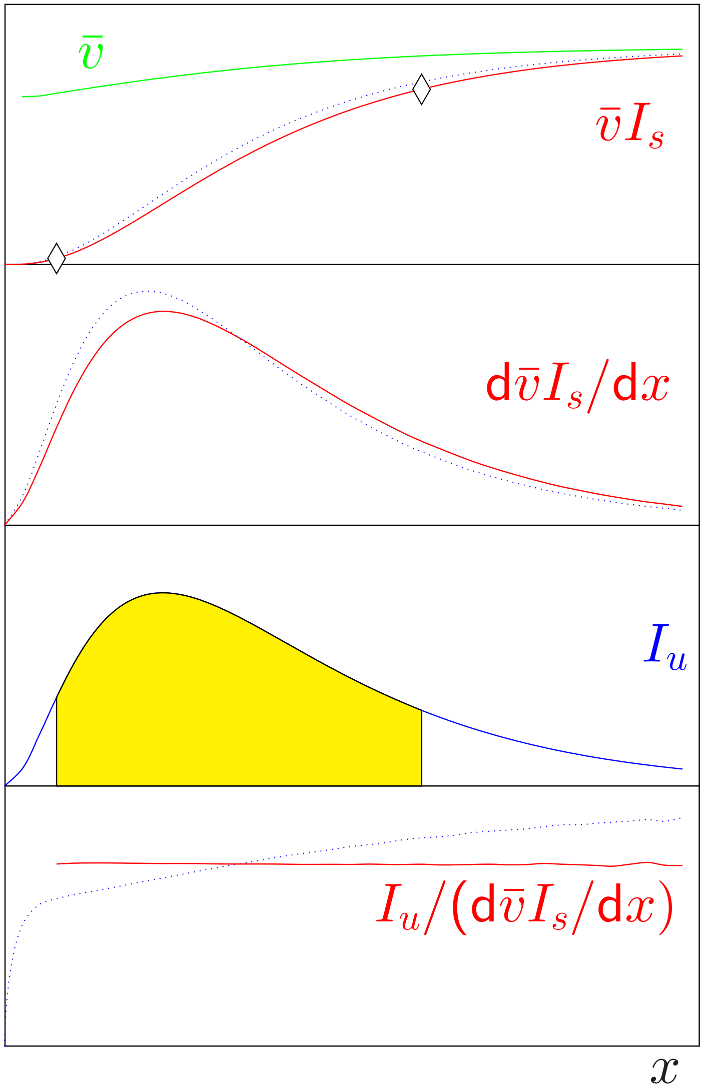
The proposed method (especially in its integral form) provides a simple and model independent analysis of RDM results, also in the extreme experimental conditions we can expect with future radiactive beams.



1 - See, *e.g.*, P. Petkov *et al.*, Nuclear Instruments and Methods in Physics Research A **431**, 208 (1999).

With the proposed method of analysis:

$$\boldsymbol{\tau} = \frac{\int_{x_1}^{x_2} I_u(x)\mathrm{d}x}{\bar{v}_x(x_2)\ I_s(x_2) - \bar{v}_x(x_1) I_s(x_1)}$$



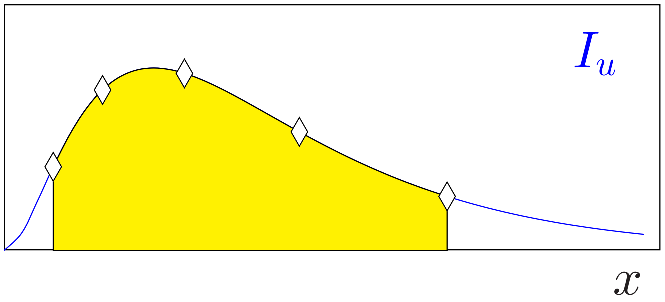
The ideal experiment:

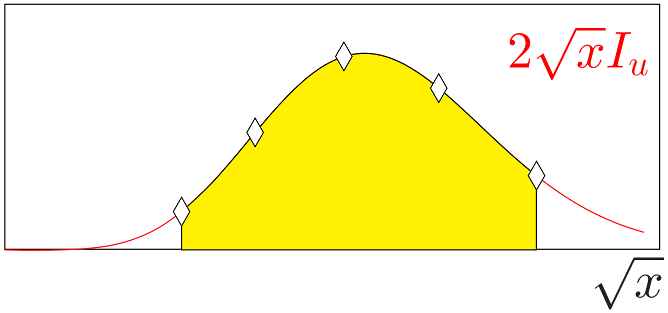
* A long measurement at x_2, to obtain I_s and $\bar{v}_x$ with good statistics (corresponding values at x_1 are much smaller and not very significant).
* A series of shorter measurements at many (regularly spaced?) points to determine the integral of I_u with comparable accuracy.


THANKS FOR YOUR ATTENTION

How can the integration be performed?

Of the continuous curve $I_u(x)$, we only know the values at a finite number of points x_n, each of which with its own statistical uncertainty σ_n.
It would be easier to work with equidistant points.
However, in the common practice the interval between two points increases with x.
We could choose, *e.g.*, $x_n = x_\circ + y^2$, with $y \propto n$
In this case, the integral can be better estimated with a change of the integration variable from x to $y = \sqrt{x - x_\circ}$:





$$\int_{x_1}^{x_2} I_u(x)\mathrm{d}x = 2\int_{y_1}^{y_2} I_u(y)y\mathrm{d}y$$

It would be preferable, in this case, that the errors on $I_u(x_n)$ decrease with x, to give $\sigma_k y_k \equiv \sigma_k \sqrt{x - x_\circ}$ approximately independent of k.