Effects of self-consistency in semiclassical pairing theory

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semiclassical approx.

Inspired by:

M. Urban& P.Schuck, Phys. Rev. **A 73**, 013621 (2006)
for atomic clusters ($A \sim 10^3 - 10^6$),
semiclassical approx. valid if

$$p_F R/\hbar >> 1,$$

is well satisfied also in medium-heavy nuclei.
Nuclei are "small" systems, charachterized by eigenfrequencies

$$\omega_0 >> \Delta/\hbar,$$

while in "large" systems

$$\omega_0 << \Delta/\hbar$$

TDHF and TDHFB

In density-matrix formulation

TDHF theory:

$$i\hbar\partial_t\rho = [h, \rho]$$
$$\rho^2 = \rho$$

TDHFB theory:

$$i\hbar\partial_t\mathcal{R} = [\mathcal{H}, \mathcal{R}]$$
$$\mathcal{R}^2 = \mathcal{R}$$

TDHFB

$$\mathcal{H} = \begin{pmatrix} \tilde{h} & \Delta \\ -\Delta^\dagger & -\tilde{h}^\dagger \end{pmatrix}$$

$$\mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa^\dagger & 1 - \rho^\dagger \end{pmatrix}$$

$\tilde{h} = h - \mu$
$h =$ HF Hamiltonian
$\Delta =$ pairing field
$\rho =$ normal density matrix
$\kappa =$pairing tensor

Wigner transf.

(keep terms up to first order in $\hbar$)

$$\rho \rightarrow \boxed{\text{WT}} \rightarrow f(\mathbf{r},\mathbf{p},t)$$

$$\mathrm{TDHF-EOM} \quad i\hbar\partial_t\rho = [h,\rho] \rightarrow \boxed{\text{WT}} \rightarrow \partial_t f = \{h,f\} \text{ Vlasov eq.}$$

$$\mathrm{TDHFB-EOM} \quad i\hbar\partial_t\mathcal{R} = [\mathcal{H},\mathcal{R}] \rightarrow \boxed{\text{WT}} \rightarrow \text{four coupled eqs. for}$$

$$f(\mathbf{r},\mathbf{p},t),\ f(\mathbf{r},-\mathbf{p},t),\ \kappa(\mathbf{r},\mathbf{p},t),\ \Delta(\mathbf{r},\mathbf{p},t)$$

$f(\mathbf{r},\pm\mathbf{p},t)$ real, but $\kappa(\mathbf{r},\mathbf{p},t)$ and $\Delta(\mathbf{r},\mathbf{p},t)$ are complex,
four eqs. and six unknowns

in TDHFB theory $\Delta(\mathbf{r},\mathbf{p},t)$ is related to $\kappa(\mathbf{r},\mathbf{p},t)$ by self-consistency

self-consistency

we assume that $\Delta(\mathbf{r},\mathbf{p},t) \approx \Delta(\mathbf{r},t)$,
and use gap eq. written in the form

$$g \int d\mathbf{p} \Big(\frac{\kappa(\mathbf{r},\mathbf{p},t)}{\Delta(\mathbf{r},t)} + \frac{1}{p^2/m} \Big) = 1$$

Since we want to study linearized EOM, we derive the two linearized s.-c. relations

$$\begin{aligned} \int d\mathbf{p} \Big(\delta\kappa^r(\mathbf{r},\mathbf{p},t) - \kappa_0(\mathbf{r},\mathbf{p}) \frac{\delta\Delta^r(\mathbf{r},t)}{\Delta_0(\mathbf{r})} \Big) &= 0 \\ \int d\mathbf{p} \Big(\delta\kappa^i(\mathbf{r},\mathbf{p},t) - \kappa_0(\mathbf{r},\mathbf{p}) \frac{\delta\Delta^i(\mathbf{r},t)}{\Delta_0(\mathbf{r})} \Big) &= 0 \end{aligned}$$

to close the system.

Summarizing...

- six unknown functions
- four EOM + two integral relations (from gap eq.)

closed but complicated!

Try to simplify:

- in equilibrium
  $\Delta_0(\mathbf{r},\mathbf{p}) \approx \Delta\,, phenomenological \quad parameter$
- linearized EOM ($\Rightarrow$ 4 linear diff. eqs.)
- use action-angle variables $(\mathbf{\Phi},\mathbf{I})$ instead of $(\mathbf{r},\mathbf{p})$
  (four algebraic eqs. instead of four diff. eqs.)
- approximation for s.-c. integral relations

not enough!

Not enough because self-consistency relations are still complicated.
Would like to reduce the problem to a set of coupled algebraic eqs.
Several degrees of approximation are possible:

- "brutal" approx.: $\delta\Delta(\mathbf{r}, t) = 0$, leads to problems with particle-number conservation and EWSR. In [1] this problem was solved with a *prescription*.
- better approx.: use weaker self-consistency conditions plus reasonable assumption that

$$\delta\Delta(\mathbf{r}, \omega) \propto \delta h(\mathbf{r}, \omega)$$

[1] V.I.Abrosimov *et al.*, Nucl.Phys. **A 800** (2008), 1

weaker s.-c. conditions

Original S.-c. conditions (Fourier transformed in time)

$$\int d\mathbf{p}\Big(\delta\kappa^r(\mathbf{r},\mathbf{p},\omega) - \kappa_0(\mathbf{r},\mathbf{p})\frac{\delta\Delta^r(\mathbf{r},\omega)}{\Delta_0(\mathbf{r})}\Big) = 0$$

$$\int d\mathbf{p}\Big(\delta\kappa^i(\mathbf{r},\mathbf{p},\omega) - \kappa_0(\mathbf{r},\mathbf{p})\frac{\delta\Delta^i(\mathbf{r},\omega)}{\Delta_0(\mathbf{r})}\Big) = 0$$

imply weaker conditions

$$\int d\mathbf{r}\int d\mathbf{p}\Big(\delta\kappa^r(\mathbf{r},\mathbf{p},\omega) - \kappa_0(\mathbf{r},\mathbf{p})\frac{\delta\Delta^r(\mathbf{r},\omega)}{\Delta_0(\mathbf{r})}\Big) = 0$$

$$\int d\mathbf{r}\int d\mathbf{p}\Big(\delta\kappa^i(\mathbf{r},\mathbf{p},\omega) - \kappa_0(\mathbf{r},\mathbf{p})\frac{\delta\Delta^i(\mathbf{r},\omega)}{\Delta_0(\mathbf{r})}\Big) = 0$$

+ reasonable assumption

Under action of an external driving field

$$\delta V^{ext}(\mathbf{r},\omega) = \eta Q(\mathbf{r})$$

$$h(\mathbf{r},\mathbf{p},\omega) = h_0(\mathbf{r},\mathbf{p}) + \delta h(\mathbf{r},\omega)$$

$\delta h(r,\omega)$ can be treated in different approximations:

- $\delta h(\mathbf{r},\omega) \approx \eta Q(\mathbf{r})$ (zero-order approx.)
- $\delta h(\mathbf{r},\omega) = \delta V^{ext}(\mathbf{r},\omega) + \delta V^{int}(\mathbf{r},\omega)$ (self-cons.)

In zero-order approx., assume that

$$\delta\Delta^{r,i}(\mathbf{r},\omega) = F^{r,i}(\omega) Q(\mathbf{r})$$

solution

$$
\begin{aligned}
F^r(\omega) &= 0\,, \\
F^i(\omega) &= \eta \frac{2\Delta}{i\omega}\,,
\end{aligned}
$$

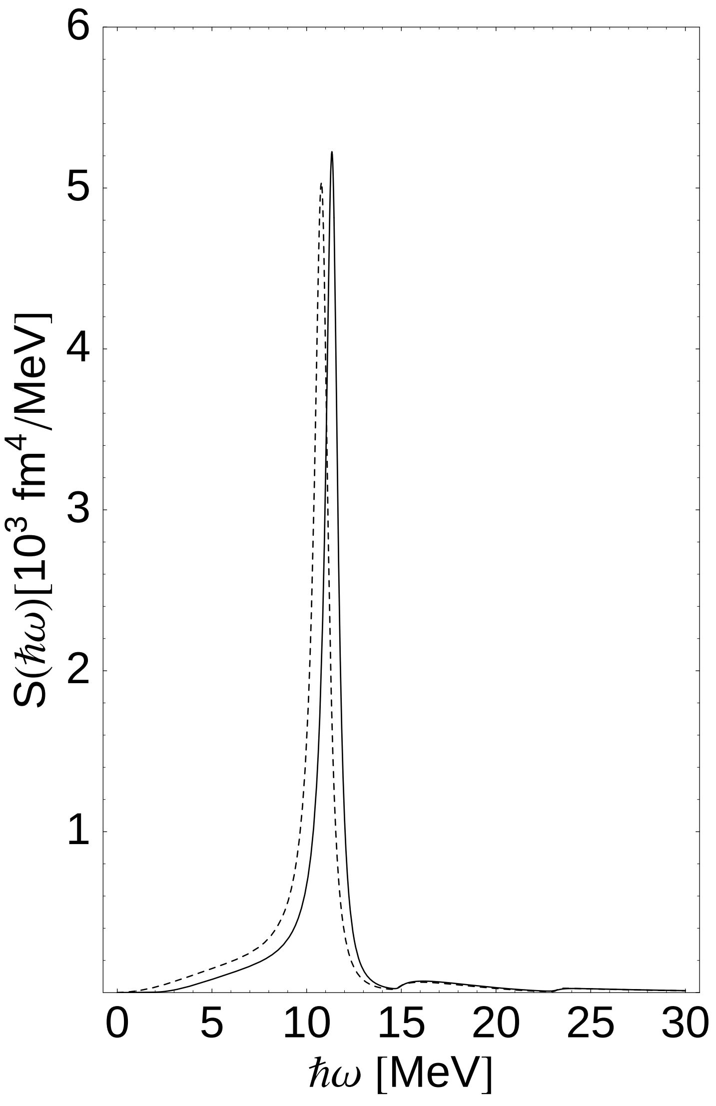
or

$$
\begin{aligned}
\delta\Delta^r(\mathbf{r},\omega) &= 0 \\
\delta\Delta^i(\mathbf{r},\omega) &= \eta \frac{2\Delta}{i\omega} Q(\mathbf{r})
\end{aligned}
$$

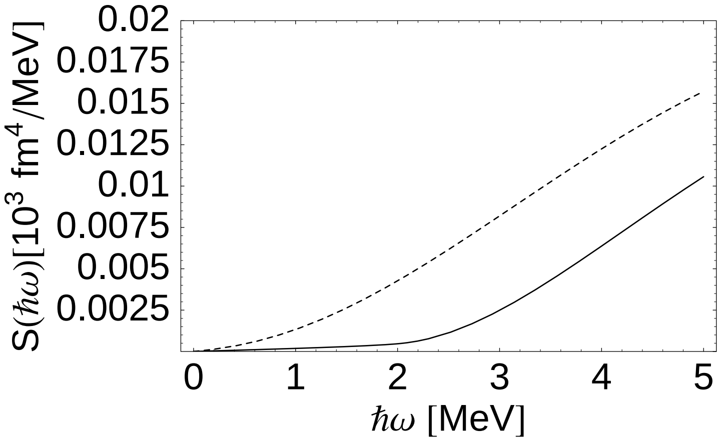
No collective pairing effects in this approx., but restoration of particle-number symmetry and correct EWSR.
HOPE to do better in near future!

giant quadrupole res.-A≈208

collective resp.



low-energy ($0\hbar\omega$) quadrup. states



giant octupole res.-A≈208

collective resp.