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Change of shell structure in nuclei towards neutron drip line

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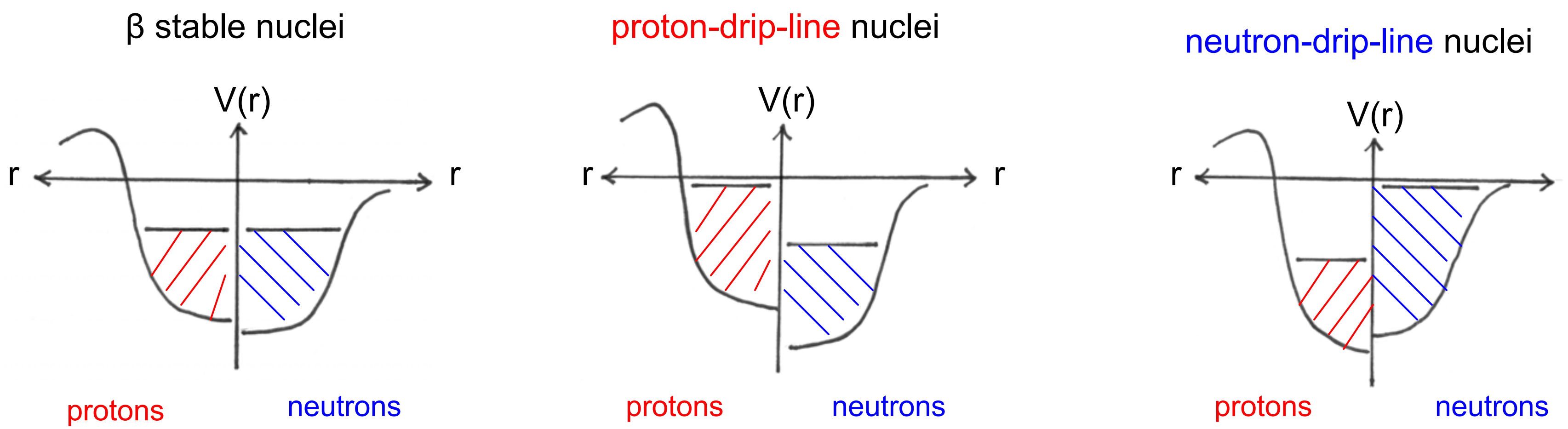
drip-line nuclei —— very different *N/Z* ratio, compared to stable nuclei with the same *A*



The study of one-particle motion in deformed potentials is the basis for understanding the structure of deformed drip-line nuclei.

The Fermi level of drip-line nuclei lies close to the continuum

$\rightarrow$ Both weakly-bound and positive-energy one-particle levels play a crucial role in the many-body correlation of those nuclei.

Change of shell structure of weakly-bound and resonant one-particle levels

Weakly-bound one-proton motion in medium-heavy nuclei may not be so different from the well-bound one, due to the high Coulomb barrier.

Spherical nuclei : unique behavior of low-ℓ orbits, as $\varepsilon_\ell\,(<0) \rightarrow 0$

Energies of neutron orbits in Woods-Saxon potential
($R = r_0 A^{1/3}$ with $r_0 = 1.27\ fm$ is varied.)

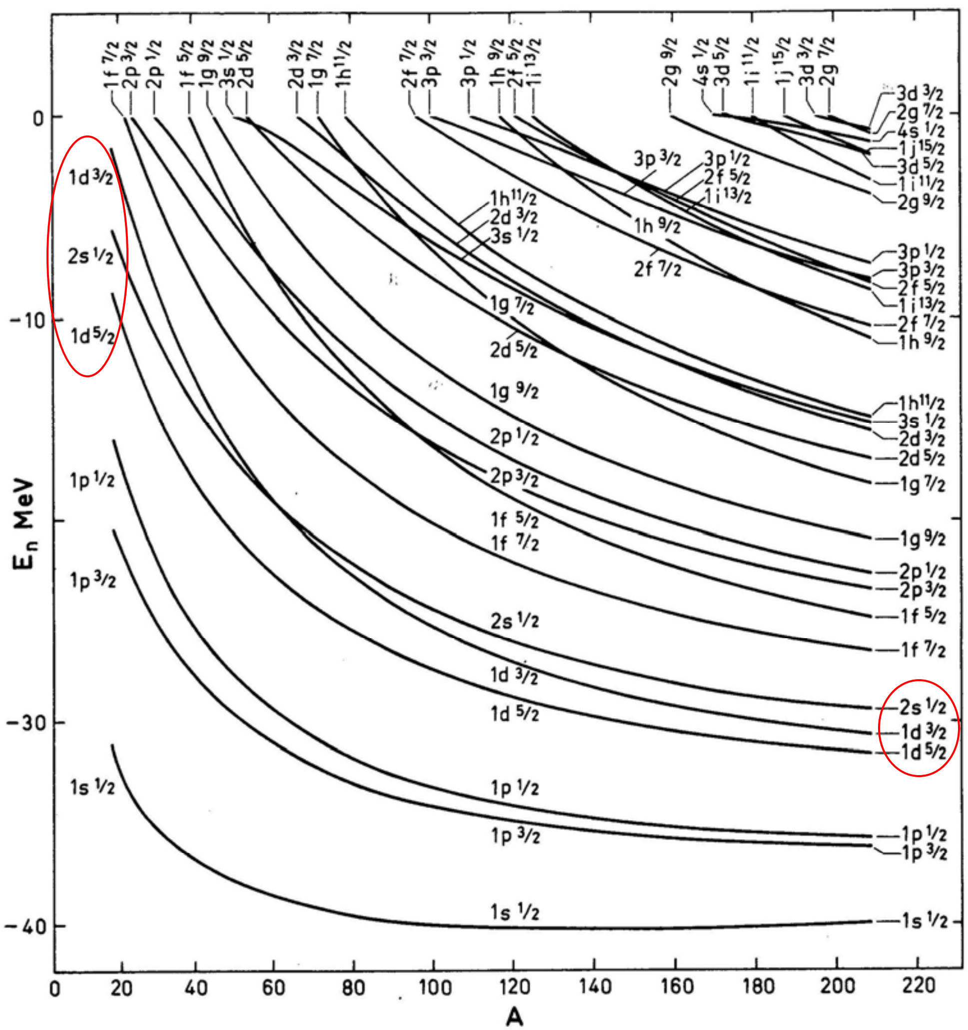
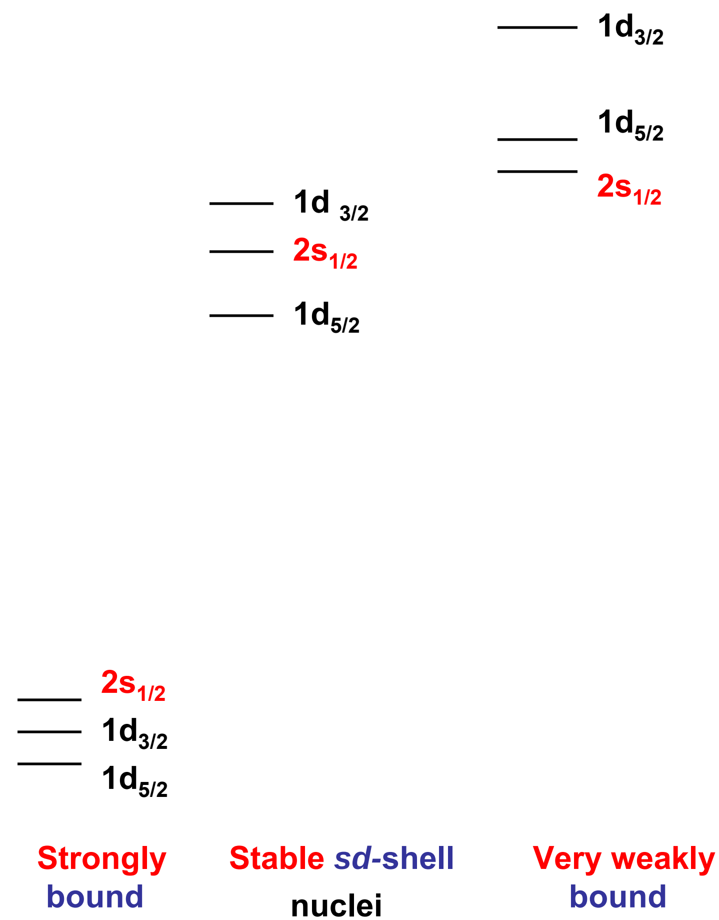


Figure 2-30 Energies of neutron orbits calculated by C. J. Veje (private communication).

Change of shell structure



in finite-well potential

Bohr & Mottelson, Nuclear Structure, Vol.II

Centrifugal potential comes from kinetic energy —— dependence on ℓ

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$$

$$=-\frac{\hbar^2}{2m}\left(\frac{1}{r}\frac{\partial^2}{\partial r^2}r+\frac{1}{r^2}\left(\frac{\partial^2}{\partial\theta^2}+\cot\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right)\right)$$

$$= -\frac{\hbar^2}{2m}\left(\frac{1}{r}\frac{\partial^2}{\partial r^2}r-\underline{\frac{1}{r^2}\frac{(\vec{\ell})^2}{\hbar^2}}\right)$$

centrifugal potential

Barrier height of [centrifugal + Woods-Saxon] potentials $\propto \frac{\ell(\ell+1)}{R_h^2}$

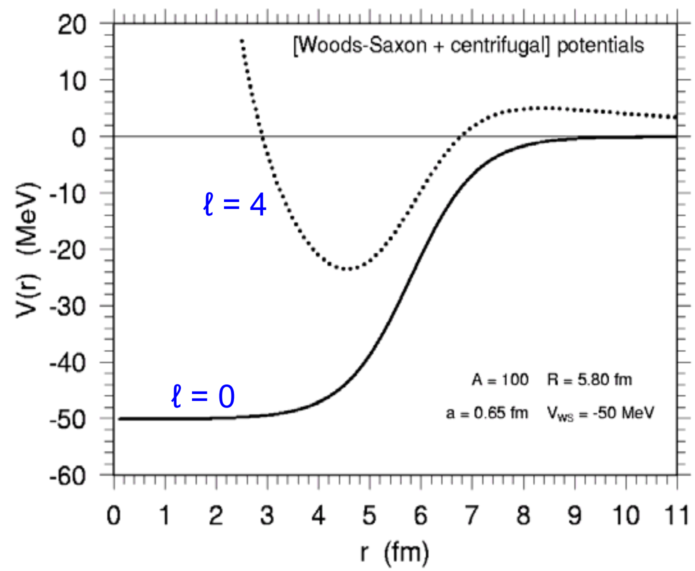
where $R_h > r_0 A^{1/3}$

No centrifugal barrier for s-orbits

$\longrightarrow$ No one-particle resonant levels for s neutrons

Spherical shape

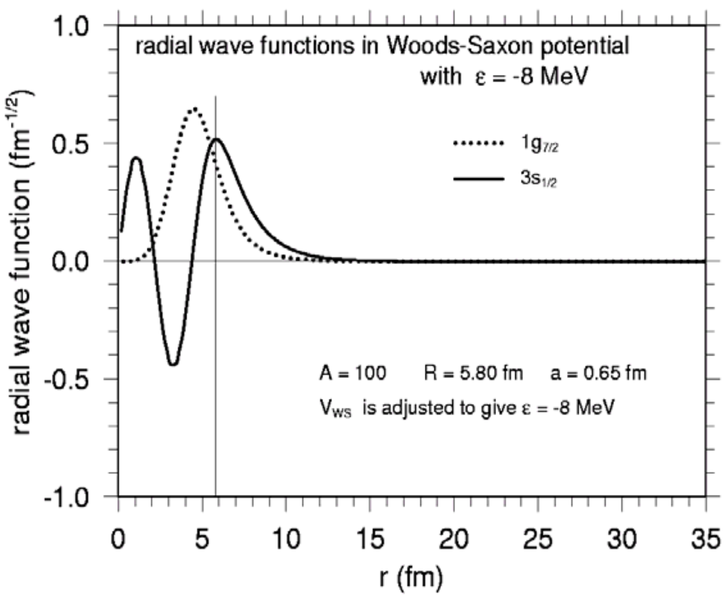
centrifugal + Woods-Saxon potentials



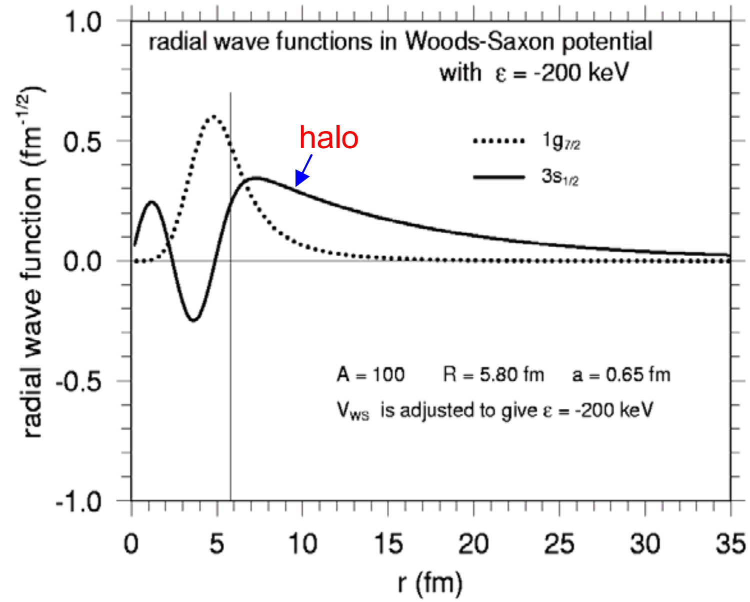
Neutron radial wave-functions

$$\Psi_{n\ell jm}(\vec{r}) = \frac{1}{r} R_{n\ell j}(r) X_{\ell jm}(\hat{r})$$

ε = – 8 MeV



ε = – 0.2 MeV



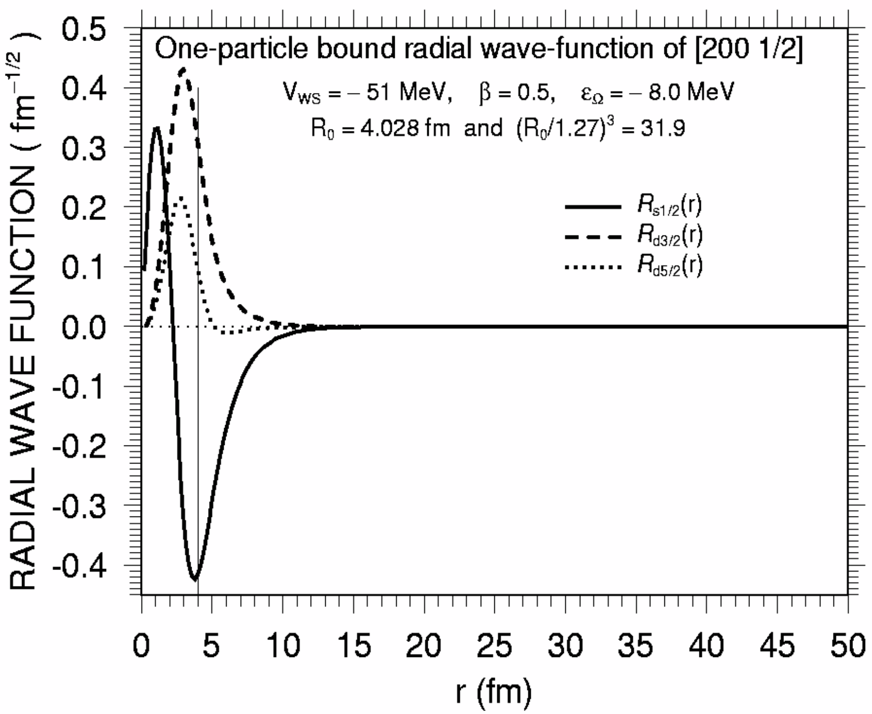
Deformed shape

Weakly-bound neutrons in deformed potential

ex. ε_Ω = -8 MeV → 0.0001 MeV for $\Omega^\pi = 1/2^+$ (i.e. various ℓ components, but $\ell_{min} = 0$)

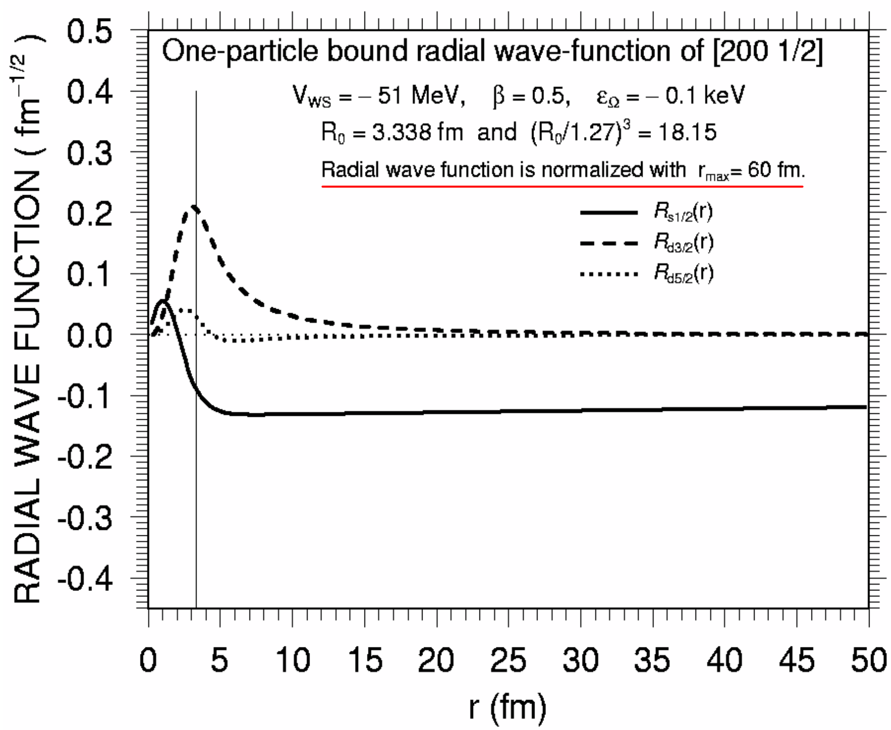
——— $s_{1/2}$	- - - - $d_{3/2}$	········ $d_{5/2}$

Bound state with ε_Ω = -8.0 MeV.



Similar behavior to wave functions in harmonic osc. potentials.

Bound state with ε_Ω = -0.0001 MeV.



Wave functions unique in finite-well potentials.

One-particle resonant levels in deformed potential

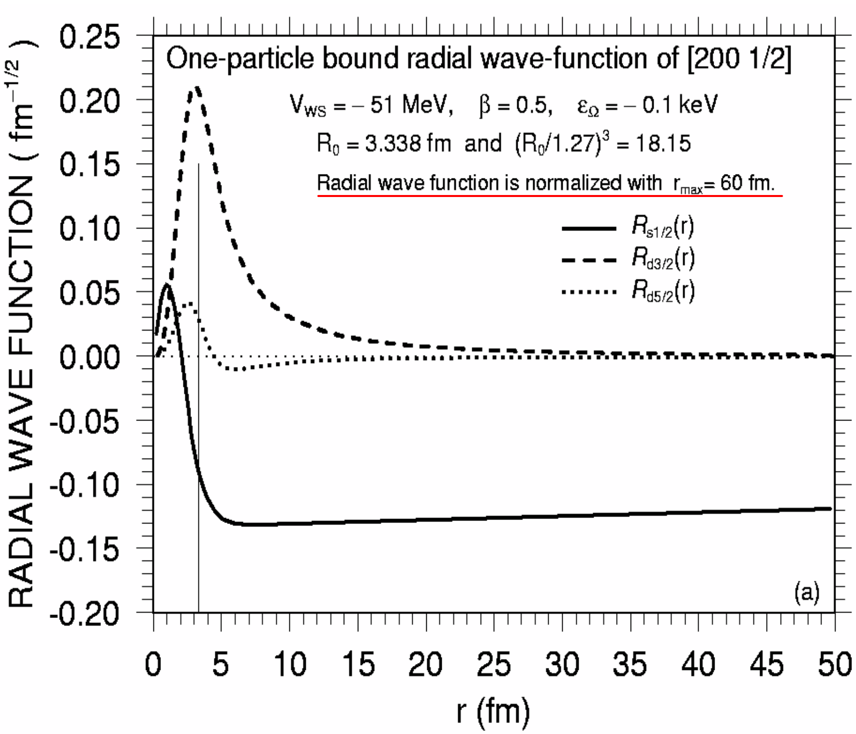
- eigenphase formalism

R.G.Newton, “Scattering Theory of Waves and Particles, (McGraw-Hill), 1966.
I.H., Phys. Rev. C **72**, 024301 (2005); **73**, 064308 (2006).

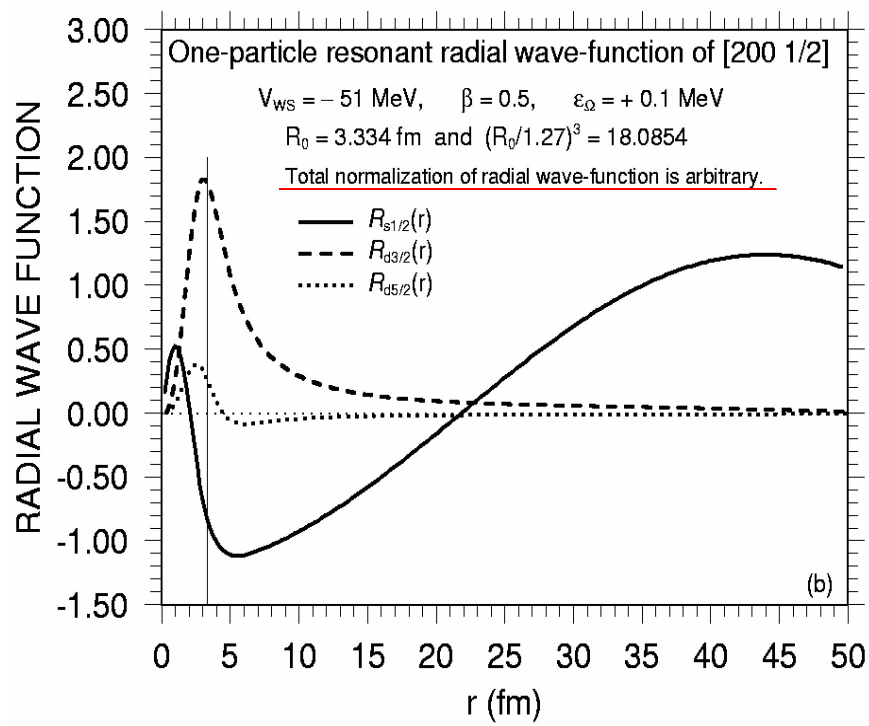
The potential radius is adjusted to obtain respective eigenvalue ($\varepsilon_\Omega < 0$) and resonance ($\varepsilon_\Omega > 0$).

—— $s_{1/2}$ - - - - $d_{3/2}$ ········ $d_{5/2}$

Bound state with ε_Ω = – 0.1 keV



Resonant state with ε_Ω = +100 keV



Existence of resonance ← d component
Width of resonance ← s component

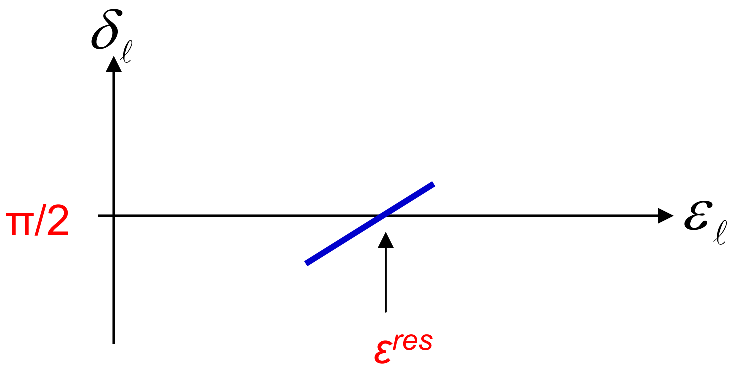
OBS. Relative amplitudes of various components inside the potential remain nearly the same for ε_Ω = – 0.1 keV → + 100 keV.

One-particle resonant level in spherical finite potentials (~~Coulomb potential~~)

For $\varepsilon_\ell > 0$ and $r \longrightarrow$ large

$$R_\ell(r) \propto \cos(\delta_\ell) krj_\ell(kr) - \sin(\delta_\ell) krn_\ell(kr) \qquad \text{where} \qquad k^2 \equiv \frac{2m}{\hbar^2}\varepsilon_\ell$$

δ_ℓ : phase shift



The width of the resonance;

$$\Gamma \equiv \frac{2}{\left.\dfrac{d\delta}{d\varepsilon}\right|_{\varepsilon=\varepsilon^{res}}}$$

The resonance energy ε^{res} is defined so that the phase shift δ_ℓ increases with energy ε as it goes through $\pi/2$ (modulo π).

For example, see ; R.G.Newton, *SCATTERING THEORY OF WAVES AND PARTICLES,* McGraw-Hill, 1966.

At ε^{res} ; (1) a sharp peak in the scattering cross section;
(2) a significant time delay in the emergence of scattered particles;
(3) the incoming wave (i.e. particles) can strongly penetrate into the system;
(4) ………..

One-particle resonance in a deformed potential – eigenphase formalism

$$H\Psi_\Omega = \varepsilon_\Omega \Psi_\Omega$$

where one-particle wave function $\Psi_\Omega(\vec{r}) = \frac{1}{r}\sum_{\ell j} R_{\ell j\Omega}(r) Y_{\ell j\Omega}(\hat{r})$

one-particle energy ε_Ω

and $Y_{\ell j\Omega}(\hat{r}) \equiv \sum_{m_\ell, m_s} C(\ell, \frac{1}{2}, j; m_\ell, m_s, \Omega) Y_{\ell m_\ell}(\hat{r}) \chi_{m_s}$

Solving the coupled differential equations derived from the Schrödinger equation with the boundary conditions,

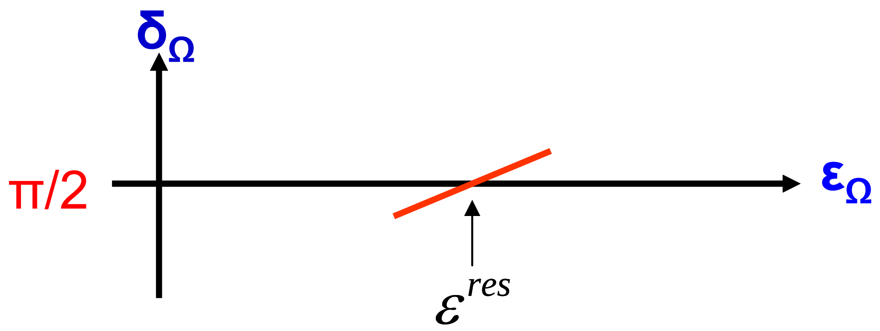
$$\begin{cases} R_{\ell j\Omega}(r) = 0 & \text{for} \quad r = 0 \\ R_{\ell j\Omega}(r) \propto \cos(\delta_\Omega) F_\ell(r) - \sin(\delta_\Omega) G_\ell(r) & \text{for} \quad r \to \infty \end{cases}$$

where $F_\ell(r) = krj_\ell(kr)$, $G_\ell(r) = krn_\ell(kr)$, $k^2 = \frac{2m}{\hbar^2}\varepsilon_\Omega$

δ_Ω : eigenphase common to all ℓj channels

A given eigenchannel : asymptotic radial wave-functions behave in the same way for all ($\ell j\Omega$) components.

One-particle resonant level in a deformed potential :
one of eigenphases δ_Ω increases through $\pi/2$ as ε_Ω increases.



When one-particle resonant level in terms of one eigenphase is obtained, the width Γ_Ω of the resonance in the intrinsic system is calculated by

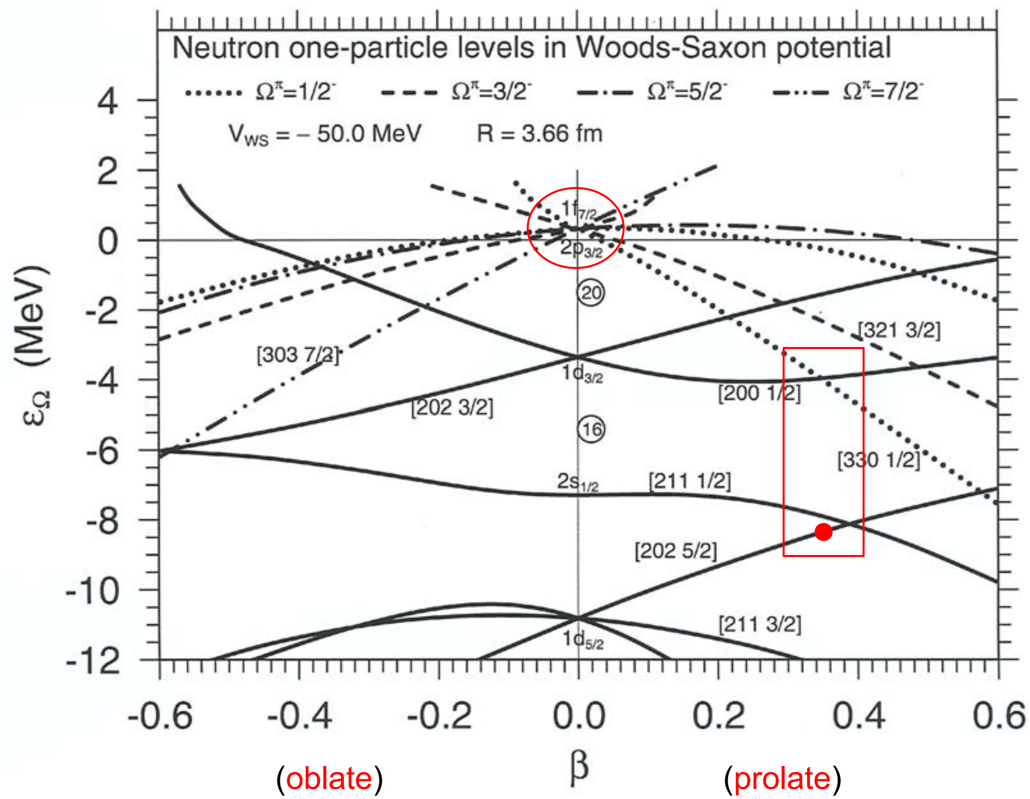
$$\Gamma_\Omega \equiv \frac{2}{\left[\dfrac{d\delta_\Omega}{d\varepsilon_\Omega}\right]_{\varepsilon_\Omega=\varepsilon_\Omega^{res}}}$$

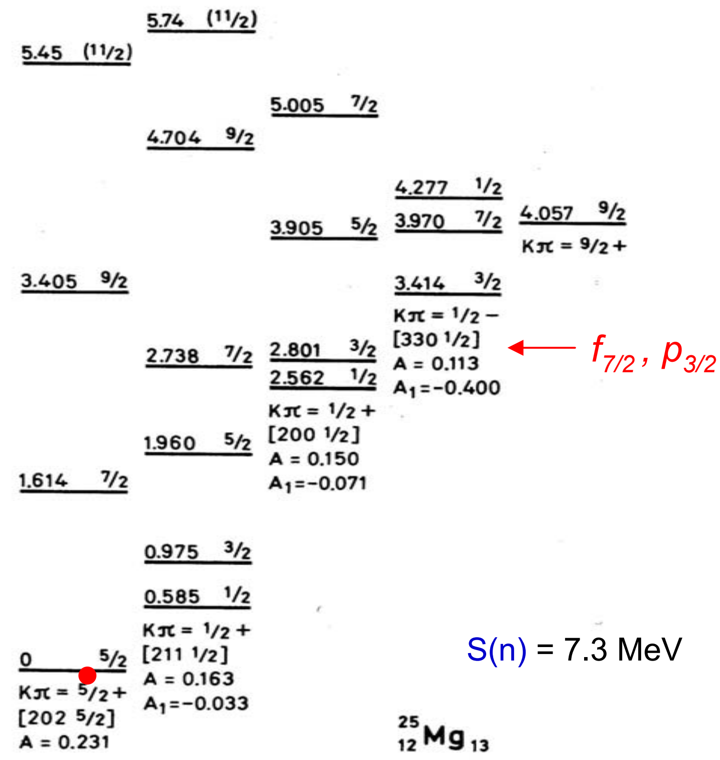
: intrinsic width

ex. The N=13 th neutron orbits observed as low-lying excitations in $^{25}Mg_{13}$ - a textbook example

$[N\ n_z\ \Lambda\ \Omega]$

Bohr & Mottelson, Nuclear Structure, Vol.II





Nilsson levels : double ($\pm\Omega$) degeneracy

$\varepsilon_{res}(f_{5/2}) = 10.4$ MeV

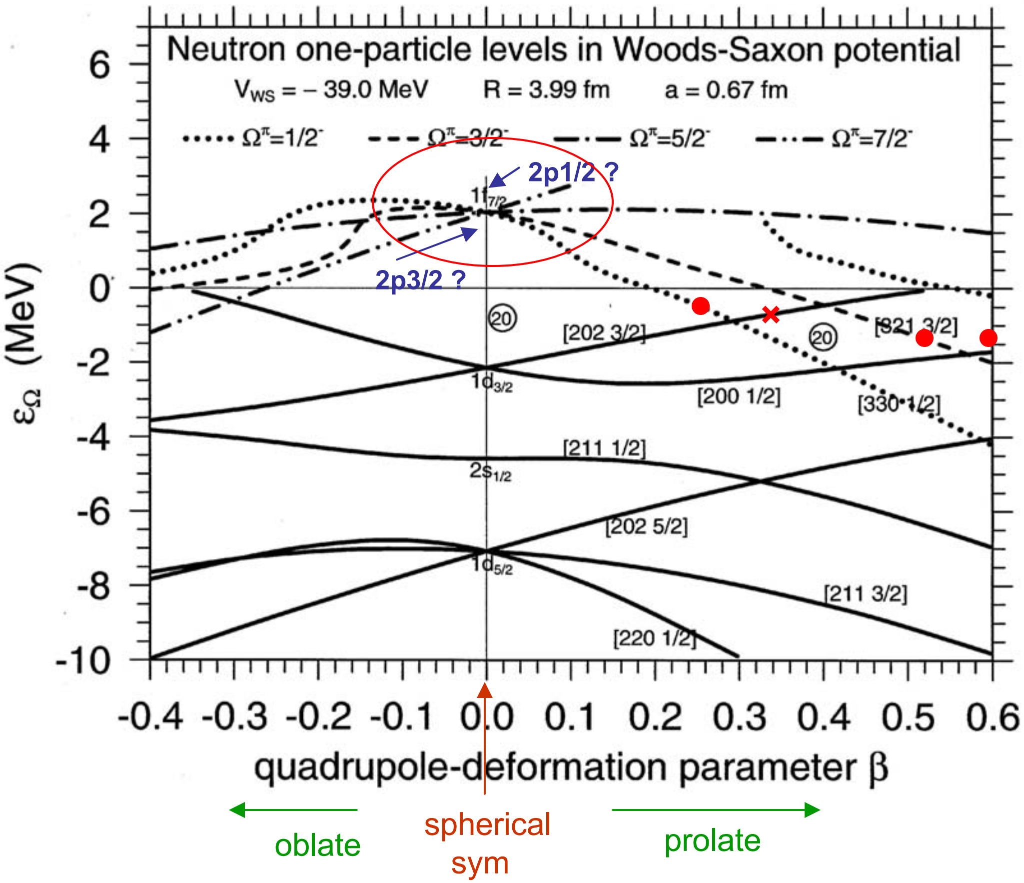
$\varepsilon_{res}(f_{7/2}) = 0.32$ MeV

$\varepsilon_{res}(p_{3/2}) = 0.31$ MeV

The above interpretation of the data works quantitatively :

- measured large E2 transitions within the bands
  $\rightarrow \quad \beta \approx 0.4$
- observed E2- and M1-intensity relations
  $\rightarrow \quad g_s^{eff} = (0.7 - 0.9)\ g_s^{free}$

One-particle neutron energies as a function of quadrupole deformation β



$[N\ n_z\ \Lambda\ \Omega]$

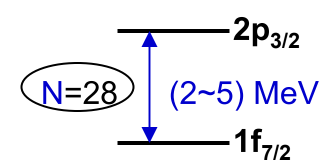
Ω : angular momentum component along the sym axis

● $^{31}Ne_{21}$ S(n) = 0.29 ± 1.64 MeV

T.Nakamura et al., PRL **103**, 262501 (2009), Coulomb breakup of 31Ne → halo structure

I.H., PRC **81**, 021304(R) (2010)

From observed properties of $^{40}_{20}Ca_{20}$ and $^{48}_{20}Ca_{28}$ (stable doubly -magic nuclei !)



Near degeneracy of $f_{7/2}$, $p_{3/2}$ and $p_{1/2}$ resonant levels can be the origin of deformed shape of those N ≈ 20 nuclei.

N = 21 neutron I^π =

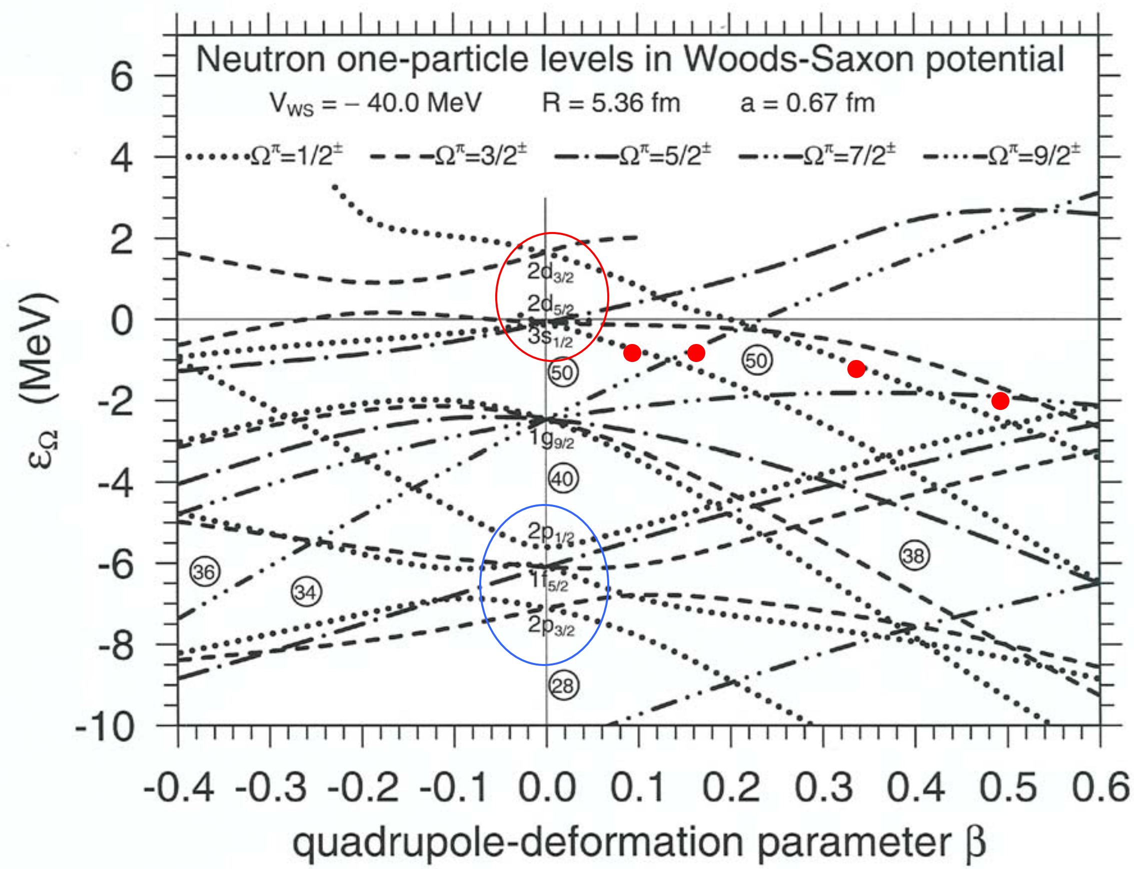
$3/2^-$	from	[330 1/2]	for	$0.20 < \beta < 0.30$	← $p(\ell=1)$ halo	●
$3/2^+$	from	[202 3/2]	for	$0.30 < \beta < 0.40$	← no halo ($\because \ell_{min} = 2$)	×
$3/2^-$	from	[321 3/ 2]	for	$0.40 < \beta < 0.58$	← $p(\ell=1)$ halo	●
$1/2^+$	from	[200 1/2]	for	$\beta > 0.58$	← $s(\ell=0)$ halo	●

One-particle neutron energies as a function of quadrupole deformation β

N ~ 50 region

A ~ 75 region

$\varepsilon_{res}(1h_{11/2}) = +5.48$ MeV
$\varepsilon_{res}(1g_{7/2}) = +3.44$ MeV
at β=0



In the case of very weak binding

At β=0 ;
$\varepsilon(3s_{1/2}) < \varepsilon(2d_{5/2})$

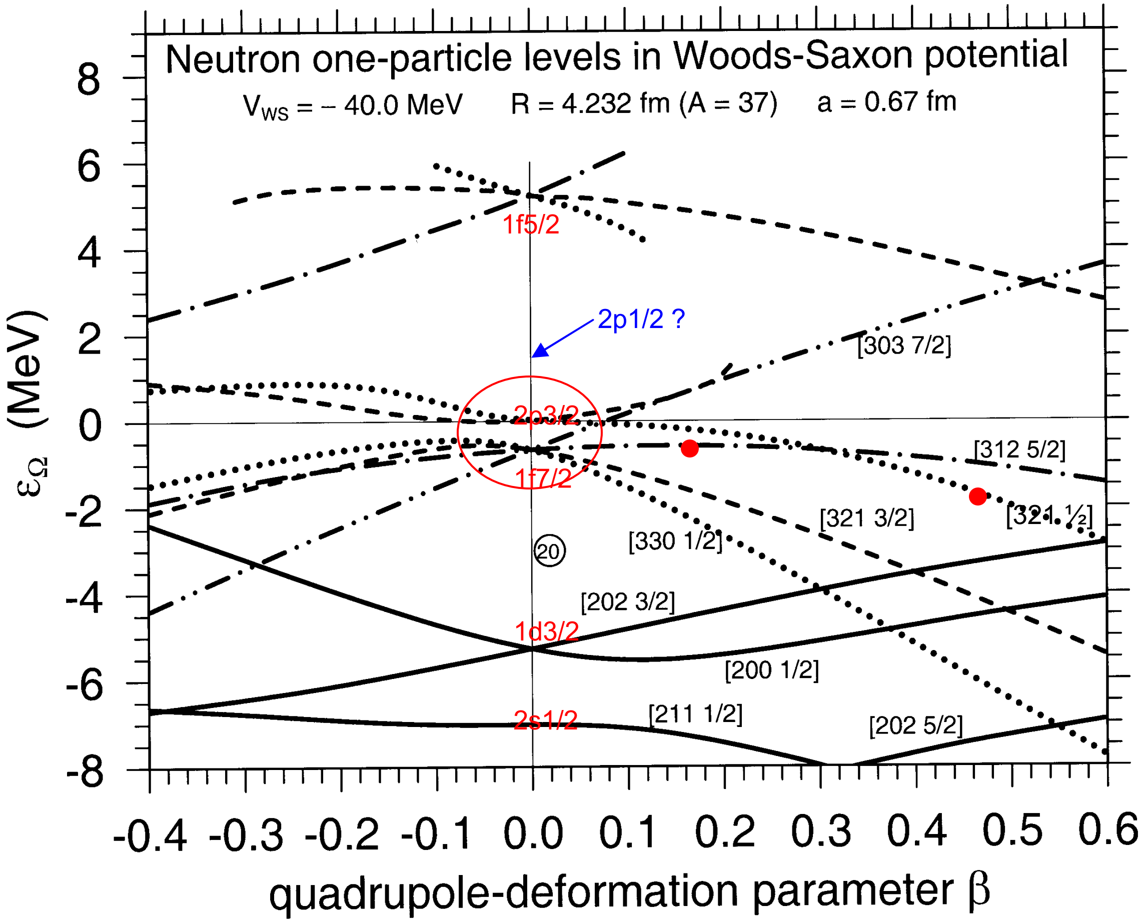
● 51^{st} neutron

ex. ${}^{75}_{24}Cr_{51}$?

Neutron-drip-line nuclei with N=51have a good chance to have the ground or very low-lying $I^{\pi} = 1/2^{+}$ state, irrespective of spherical or deformed shape.

One-particle neutron energies as a function of quadrupole deformation β

[N n_z Λ Ω]



At β=0 ;
$\varepsilon(2p_{3/2}) - \varepsilon(1f_{7/2})$
= 680 keV

● $^{37}Mg_{25}$ S(n) = a few hundreds keV ?

$I^\pi = \begin{cases} 5/2^- \text{ from } [312\ 5/2] \text{ for } 0 < \beta < 0.3 \\ 1/2^- \text{ from } [321\ 1/2] \text{ for } 0.3 < \beta < 0.6 \end{cases}$

← no halo (∵ $\ell_{min} = 3$)

← $p(\ell = 1)$ halo

I.H., Phys. Rev. C73, 064308 (2006)

Some comments on eigenphase ;

1) For a given potential and a given ε_Ω
there are several (in principle, an infinite number of) solutions of eigenphase δ_Ω .

2) The number of eigenphases for a given potential and a given ε_Ω is equal to that of wave function components with different (ℓ,j) values.

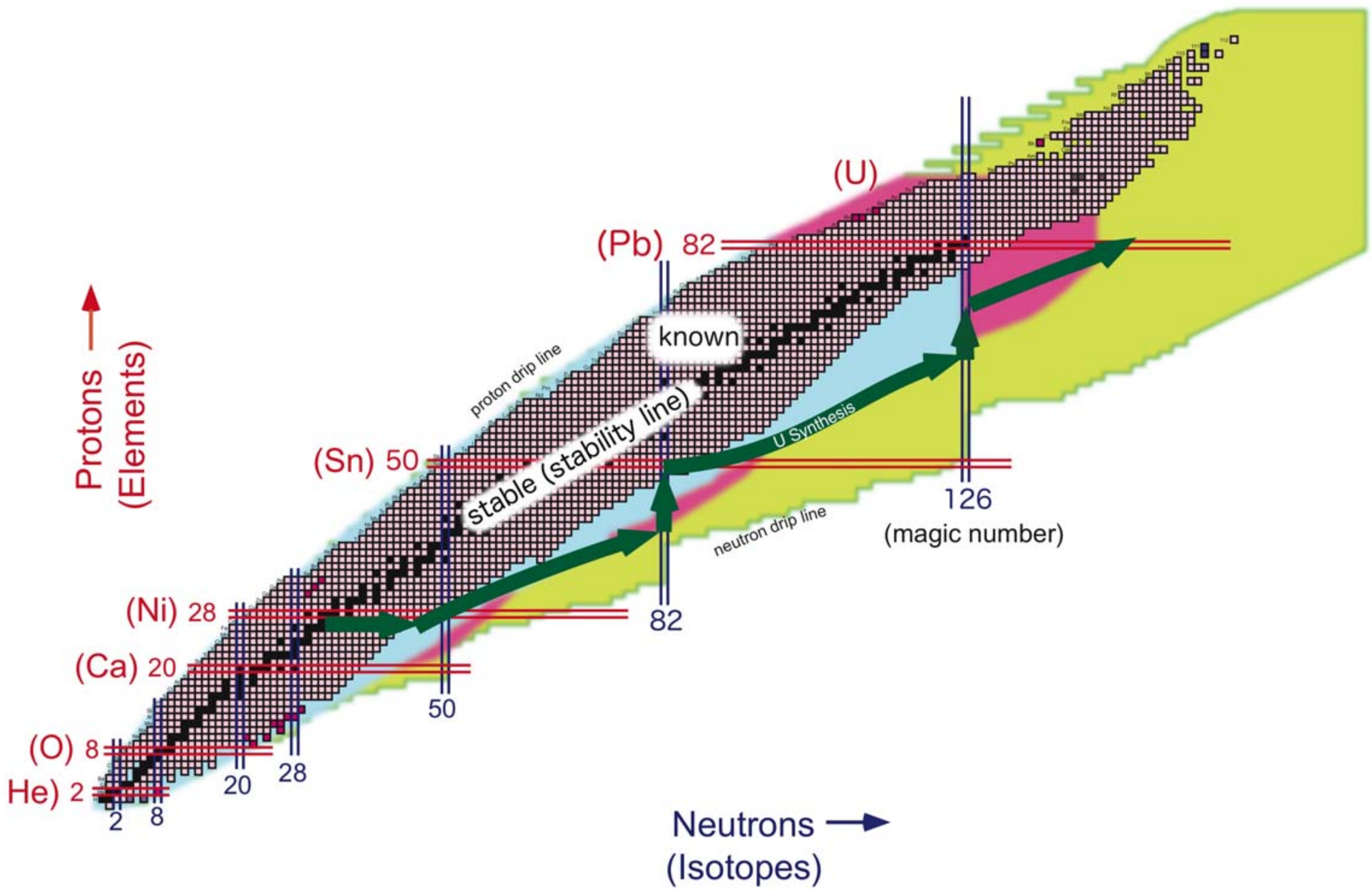
3) The value of δ_Ω determines the relative amplitudes of different (ℓ,j) components.

4) In the region of small values of ε_Ω (> 0), only one of eigenphases varies strongly as a function of ε_Ω , while other eigenphases remain close to the values of $n\pi$.

Neutron drip line [unstable against neutron emission] is known up till Z=8, ${}^{24}_{8}O_{16}$

ex. recently found nuclei inside neutron drip line ; ${}^{40}_{12}Mg_{28}$ ${}^{42}_{13}Al_{29}$ ${}^{44}_{14}Si_{30}$

Proton drip line [unstable against proton emission] is approximately known up till Pb region



Proton drip-line lies much closer to β-stability line than neutron drip-line.

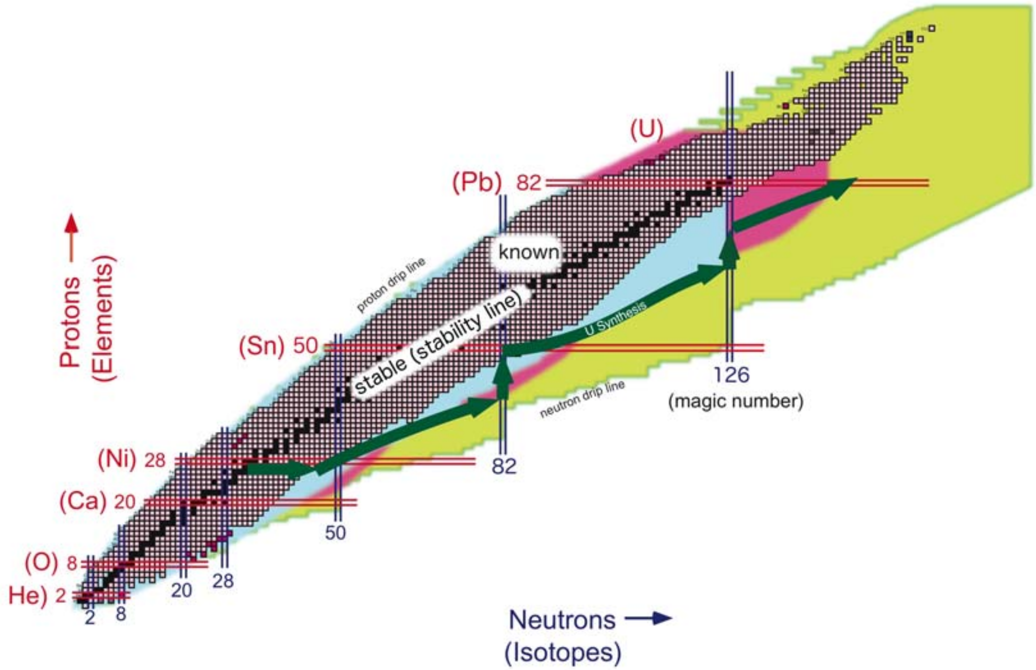
ex. *Ne* isotopes

$^{16}_{10}Ne_{6}$ —— $^{20}_{10}Ne_{10}$ —— $^{34}_{10}Ne_{24}$

proton drip-line nucleus

most abundant stable nucleus

inside the neutron drip-line



T.Motobayashi (2006)

Centrifugal potential + Woods-Saxon potential —— dependence on ℓ

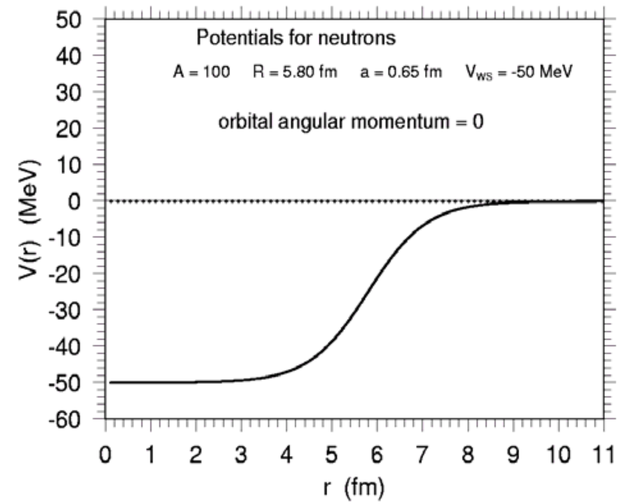
$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+V(r)$$

$$=-\frac{\hbar^2}{2m}\left(\frac{1}{r}\frac{\partial^2}{\partial r^2}r+\frac{1}{r^2}\left(\frac{\partial^2}{\partial\theta^2}+\cot\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right)\right)+V(r)$$

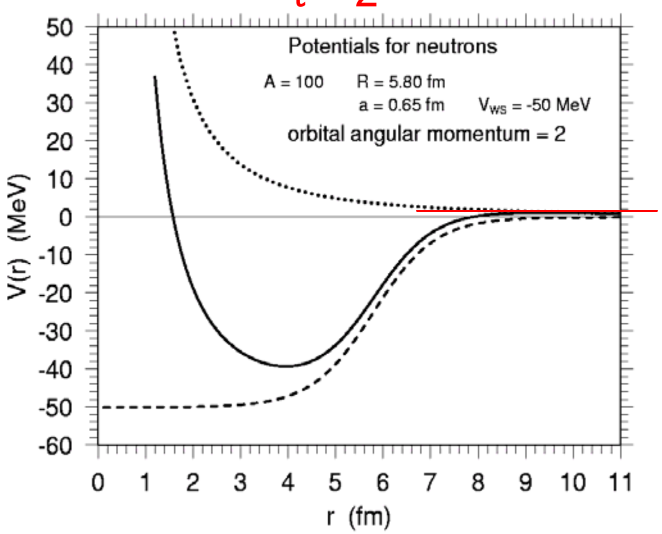
$$= -\frac{\hbar^2}{2m}\left(\frac{1}{r}\frac{\partial^2}{\partial r^2}r-\frac{1}{r^2}\frac{(\vec{\ell})^2}{\hbar^2}\right)+V(r)$$

centrifugal potential

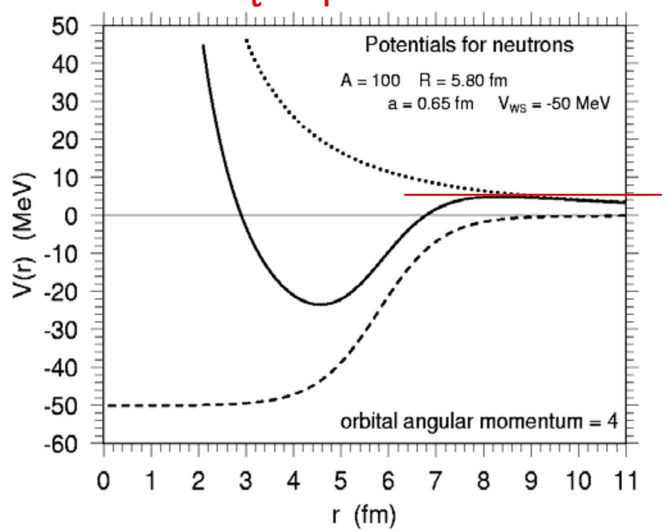
$\ell = 0$



$\ell = 2$



$\ell = 4$



- - - - Woods-Saxon pot.

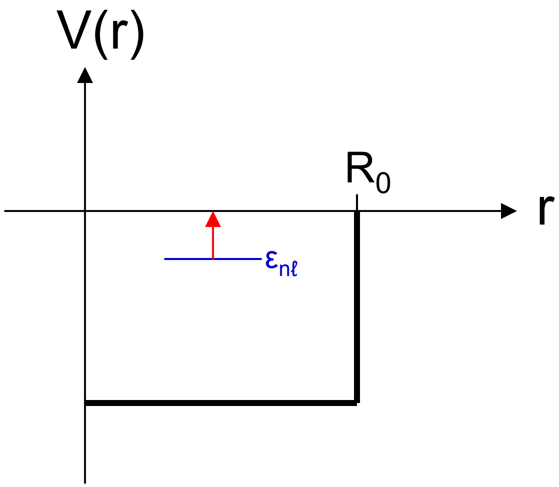
.... centrifugal pot.

—— W-S + centrifugal pot.

Barrier height $\propto \frac{\ell(\ell+1)}{R_h^2}$

where $R_h > r_0 A^{1/3}$

For a finite square-well potential



The probability for one neutron to stay inside the potential, when the eigenvalue $\varepsilon_{n\ell}$ (< 0) → 0

ℓ	0	1	2	3
$\int_0^{R_0} \lvert R_{n\ell}(r) \rvert^2 \, dr$	0	1/3	3/5	5/7

Root-mean-square radius, r_{rms}, of one neutron ; $r_{rms} \equiv \sqrt{\langle r^2 \rangle}$

In the limit of $\varepsilon_{n\ell}$ (<0) → 0

$r_{rms} \propto (-\varepsilon_{n\ell})^{-1/2} \rightarrow \infty$ for $\ell = 0$

$(-\varepsilon_{n\ell})^{-1/4} \rightarrow \infty$ for $\ell = 1$

finite value for $\ell \geq 2$

Schrödinger equation for one-particle motion with spherical finite potentials

$$H = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+V(r)+V_{\ell s}(r)$$

(x, y, z) ⟶ *(r, θ, φ)*

$$H\Psi = \varepsilon\Psi \qquad \Psi = \frac{1}{r}R_{n\ell j}(r)X_{\ell j m_j}(\hat{r})$$

where

$$X_{\ell j m_j}(\hat{r}) = \sum_{m_\ell, m_s} C(\ell,\frac{1}{2},j;m_\ell,m_s,m_j)Y_{\ell m_\ell}(\theta,\phi)\chi_{1/2,m_s}$$

$$(\vec{\ell})^2 Y_{\ell m}(\theta,\phi) = \hbar^2\ell(\ell+1)Y_{\ell m}(\theta,\phi)$$

The Shrödinger equation for radial wave-functions is written as

$$\left\{\frac{d^2}{dr^2}-\frac{\ell(\ell+1)}{r^2}+\frac{2m}{\hbar^2}\left(\varepsilon_{n\ell j}-V(r)-V_{\ell s}(r)\right)\right\}R_{n\ell j}(r)=0 \qquad (\$)$$

For example, for neutrons eq.($) should be solved with the boundary conditions;

at $r = 0$ $\qquad R_\ell(r) = 0$

at $r \to$ large (where $V(r) = 0$)

for $\varepsilon_\ell < 0$ $\qquad R_\ell(r) \propto \alpha r k_\ell(\alpha r)$ $\qquad$ where $\alpha^2 = -\frac{2m}{\hbar^2}\varepsilon_\ell$

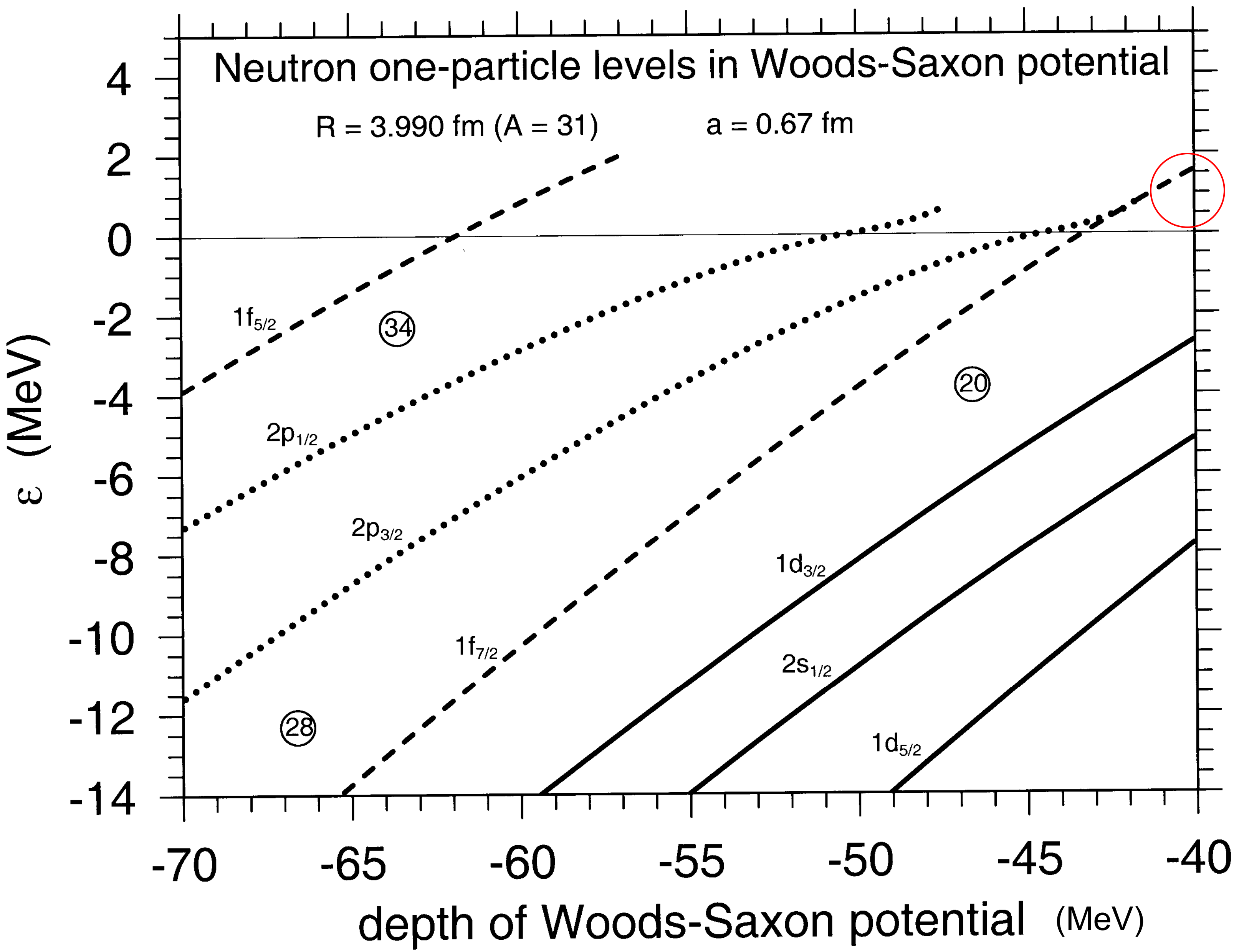
for $\varepsilon_\ell > 0$ $\qquad R_\ell(r) \propto \cos(\delta_\ell)krj_\ell(kr)-\sin(\delta_\ell)krn_\ell(kr)$ $\qquad$ where $k^2 = \frac{2m}{\hbar^2}\varepsilon_\ell$

δ_ℓ : phase shift

k_ℓ : Modified spherical Bessel function of the third kind (See p.443 of Abramowitz & Stegun, Handbook of mathematical functions)

j_ℓ : spherical Bessel function

n_ℓ : spherical Neumann function


Neutron one-particle levels in Woods-Saxon potential
R = 3.990 fm (A = 31)
a = 0.67 fm
1f5/2
34
2p1/2
2p3/2
20
1d3/2
1f7/2
2s1/2
28
1d5/2
4
2
0
-2
-4
-6
-8
-10
-12
-14
ε (MeV)
-70
-65
-60
-55
-50
-45
-40
depth of Woods-Saxon potential (MeV)