DOUBLE BETA DECAY; NUCLEAR STRUCTURE ASPECTS

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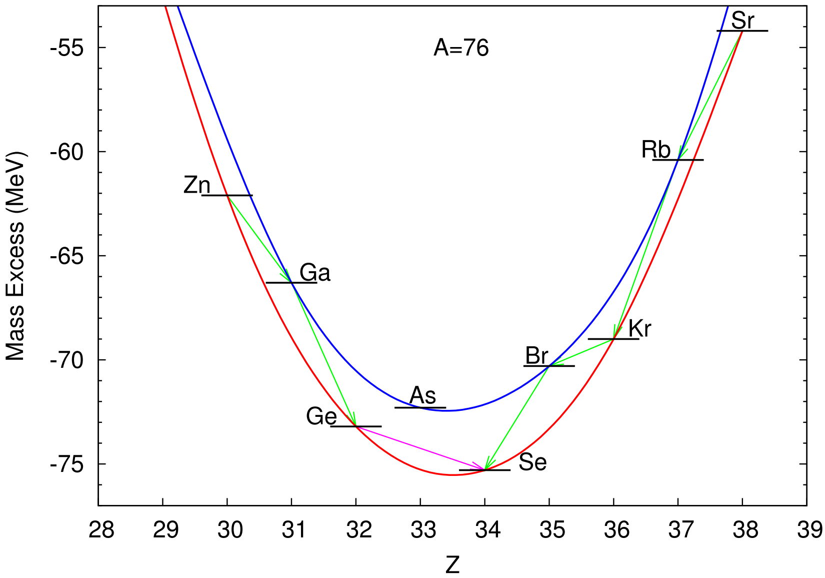
OUTLINE

- Basics.
- ISM results for the 2ν decays
- The 0ν operators.
- The nuclear wave functions
- "State of the Art" Nuclear Matrix Elements.
- The role of correlations; pairing *vs* deformation
- Conclusions.

Double beta decay

Some nuclei, otherwise nearly stable, can decay emitting two electrons and two neutrinos ($2\nu\ \beta\beta$) by a second order process mediated by the weak interaction. This decay has been experimentally measured in a few cases.

This process exists due to the **nuclear pairing interaction** that favors energetically the even-even isobars over the odd-odd ones.


A=76
Mass Excess (MeV)
-55
-60
-65
-70
-75
28
29
30
31
32
33
34
35
36
37
38
39
Z
Zn
Ga
Ge
As
Se
Br
Kr
Rb
Sr

Double beta decays

When the single beta decay to the intermediate odd-odd nucleus is forbidden, the only decay channel open is the ($2\nu\ \beta\beta$). For instance, ^{76}Ge cannot decay to ^{76}As and decays to ^{76}Se instead. The decay probability contains a phase space factor and the square of a nuclear matrix element

$$[T^{2\nu}_{1/2}]^{-1} = G_{2\nu}|M^{2\nu}_{GT}|^2$$

For ^{76}Ge, $T^{2\nu}_{1/2} = (1.7 \pm 0.2)\text{x}10^{21}$ years

The $\beta\beta$ emitters

$\beta\beta$ emitters with $Q_{\beta\beta}$ >2 Mev

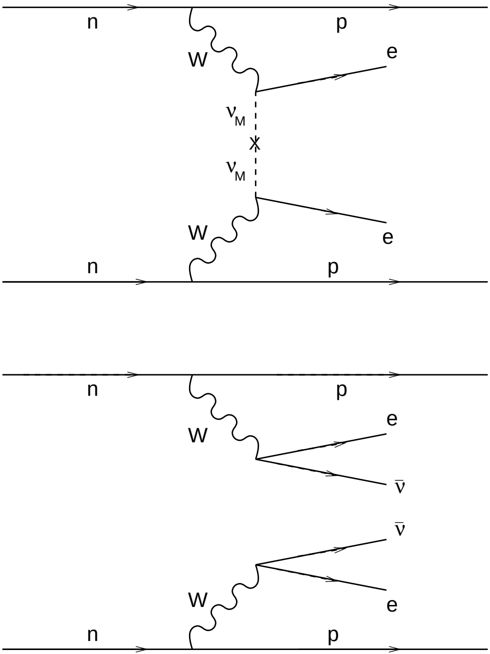
Transition	$Q_{\beta\beta}$ (keV)	Abundance (%) ($^{232}Th = 100$)
$^{110}Pd \rightarrow ^{110}Cd$	2013	12
$^{76}Ge \rightarrow ^{76}Se$	2040	8
$^{124}Sn \rightarrow ^{124}Te$	2288	6
$^{136}Xe \rightarrow ^{136}Ba$	2479	9
$^{130}Te \rightarrow ^{130}Xe$	2533	34
$^{116}Cd \rightarrow ^{116}Sn$	2802	7
$^{82}Se \rightarrow ^{82}Kr$	2995	9
$^{100}Mo \rightarrow ^{100}Ru$	3034	10
$^{96}Zr \rightarrow ^{96}Mo$	3350	3
$^{150}Nd \rightarrow ^{150}Sm$	3667	6
$^{48}Ca \rightarrow ^{48}Ti$	4271	0.2

The I(nteracting)SM predictions *vs* the experimental results for the 2ν double beta decays

$M^{(2\nu)}$ (MeV^{-1})	exp	th	INT
$^{48}Ca \rightarrow {}^{48}Ti$	0.05 ± 0.01	0.047	KB3
$^{48}Ca \rightarrow {}^{48}Ti$	0.05 ± 0.01	0.048	KB3G
$^{48}Ca \rightarrow {}^{48}Ti$	0.05 ± 0.01	0.065	GXPF1
$^{76}Ge \rightarrow {}^{76}Se$	0.13 ± 0.01	0.107	gcn28:50
$^{76}Ge \rightarrow {}^{76}Se$	0.13 ± 0.01	0.105	HOMHJ
$^{82}Se \rightarrow {}^{82}Kr$	0.10 ± 0.01	0.120	gcn28:50
$^{82}Se \rightarrow {}^{82}Kr$	0.10 ± 0.01	0.108	HOMHJ
$^{128}Te \rightarrow {}^{128}Xe$	0.05 ± 0.005	0.059	gcn50:82
$^{130}Te \rightarrow {}^{130}Xe$	0.032 ± 0.003	0.043	gcn50:82
$^{136}Xe \rightarrow {}^{136}Ba$	$<0.01\pm0.01$	0.041	gcn50:82

We include the standard quenching in the full $0\hbar\omega$ calculations (q=0.77 in the A=48 case). We fit the quenching factor for the other spaces to the GT single beta decays of the region, getting q=0.56 instead of the standard q=0.7 value

If the neutrinos are massive Majorana particles, the double beta decay can also take place without emission of neutrinos ($0\nu\ \beta\beta$).


n
p
W
e
ν_M
X
ν_M
W
e
n
p
n
p
W
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$\bar{\nu}$
$\bar{\nu}$
W
e
n
p

Has the neutrinoless double beta decay been observed?

There is an unconfirmed claim of discovery by (part of) the Heildelberg-Moscow collaboration (Klapdor 2001, 2004) of the $^{76}Ge \rightarrow ^{76} Se$ neutrinoless decay with a half-life of 2.2 x 10^{25} years

The neutrinoless double beta decay

The expression for the neutrinoless beta decay half-life, in the mass mode, for the $0^+ \rightarrow 0^+$ decay, can be brought to the following form:

$$[T_{1/2}^{(0\nu)}(0^+ - > 0^+)]^{-1} = G_{0\nu}\left(M^{(0\nu)}\left(\frac{\langle m_\nu \rangle}{m_e}\right)\right)^2$$

$G_{0\nu}$ is the kinematic phase space factor, $M^{0\nu}$ the nuclear matrix element (NME) that has Fermi, Gamow-Teller and Tensor contributions, and $\langle m_\nu \rangle$ the effective neutrino mass.

The neutrinoless double beta decay

$$M^{(0\nu)} = \left(\frac{g_A}{1.25}\right)^2 \left(M_{GT}^{(0\nu)} - \frac{M_F^{(0\nu)}}{g_A^2} - M_T^{(0\nu)}\right)$$

$$\langle m_\nu \rangle = \sum_k U_{ek}^2 m_k$$

The U's are the matrix elements of the weak mixing matrix.

The Nuclear Matrix Elements

The matrix elements $\mathrm{M}^{(0\nu)}_{GT,F,T}$ can be written as,

$$M_K^{(0\nu)} = \langle 0_f^+ | H_K(|\vec{r_1} - \vec{r_2}|)(t_1^- \; t_2^-)\Omega_K | 0_i^+ \rangle$$

$$\text{with} \;\; \Omega_F = 1, \; \Omega_{GT} = \vec{\sigma}_1 \cdot \vec{\sigma}_2, \; \Omega_T = S_{12}$$

$H_K(|\vec{r_1} - \vec{r_2}|)$ are the neutrino potentials ($\sim$1/r) obtained from the neutrino propagator.

The Nuclear Matrix Elements: Closure

The neutrino potentials depend explicitly on the excitation energy of the states of the intermediate nucleus E_m. However, due to the large average energy of the virtual neutrino ($>$100 MeV), they can as well be calculated in the closure approximation, that is good to better than 90%.

Finite Size and Short Range Correlations

- The nucleon finite size is included by means of a dipole form factor
- The short range correlations, were most often taken into account by means of the Jastrow ansatz of Miller and Spencer, but softer prescriptions, as the UCOM (Unitary Correlation Operator Method) have been implemented as well. Actually, recent microscopic calculations (Brueckner-type) of the effect of the short range correlations show that, once the form factor included, the short range correlations are negligible

Quenching

- The Gamow-Teller operator needs to be quenched in the 2ν mode and in the single beta decays. However, the situation is different in the 0ν decay, because in this mode the contribution of the 1^+ channel is just one among many, it is never dominant, and often comes with opposite phase to all the others

Two main approaches have been traditionally used for the description of the nuclei involved in the transition. The Shell Model with configuration mixing in large valence spaces (ISM) and the Quasi-particle RPA in a spherical basis. More recently, the IBM (Interacting Boson Model) approach and the PHFB approximation using schematic interactions in ISM-like valence spaces, have been applied as well.

The Nuclear Wave Functions

To assess the validity of the wave functions, quality indicators are needed such as:

- Good spectroscopy for parent, daughter and grand-daughter, even better if it extends to a full mass region
- Occupancies
- GT-strengths and strength functions, 2ν matrix elements, etc.

This quality control should be applied on a decay by decay basis, because a given approach may work well for some cases and not for others.

Interacting Shell Model calculations (ISM) *vs* QRPA

- Interaction
  - ISM: Monopole corrected G-matrices
  - QRPA: Realistic or schematic interactions. In the last generation of QRPA calculations, the global strength parameters of the interaction, g_{ph} and g_{pp}, are adjusted to the experimental 2ν half-lives
- Valence space
  - ISM: A limited number of orbits, but all the possible ways of distributing the valence particles among the valence orbits are taken into account.
  - QRPA: A larger number of orbits, but a much more limited set of configurations

ISM *vs* QRPA calculations

- Pairing Correlations
  - ISM: Are treated exactly in the valence space. Proton and neutron numbers are exactly conserved. Proton-proton, neutron-neutron, and proton-neutron (isovector and isoscalar) pairing is included
  - QRPA: Only proton-proton and neutron-neutron pairing are considered. They are treated in the BCS approximation. Proton and neutron numbers are not exactly conserved
- Multipole Correlations and Deformation
  - ISM: Are described properly in the laboratory frame. Angular momentum conservation preserved
  - QRPA: The correlations are treated at the RPA level. Permanent deformation is not incorporated

The Valence Spaces

Miscellanea of computationally accessible valence spaces relevant for the description of double beta decay emitters:

(note: in a major HO shell of principal quantum number **p** the orbit j=p+1/2 is called *intruder* and the remaining ones are denoted by r_p)

- The *pf* shell; ^{48}Ca
- r_3-$g_{9/2}$: ^{76}Ge, ^{82}Se,
- r_3-$g_{9/2}$ for protons and r_4-$h_{11/2}$ for neutrons; ^{96}Zr, ^{100}Mo
- r_4-$h_{11/2}$ for neutrons and $p_{1/2}$-$g_{9/2}$-r_4 for protons: ^{110}Pd, ^{116}Cd
- r_4-$h_{11/2}$ for neutrons and protons: ^{124}Sn, $^{128-130}$Te, ^{136}Xe

The Strasbourg-Madrid codes can deal with problems involving basis of $O(10^{11})$ Slater determinants, using "relatively" modest computational resources

Update of the ISM 0ν results: UCOM SRC

	$M^{(0\nu)}$	$\langle m_\nu \rangle$
$^{48}Ca \rightarrow {}^{48}Ti$	0.85	0.63
$^{76}Ge \rightarrow {}^{76}Se$	2.81	0.72
$^{82}Se \rightarrow {}^{82}Kr$	2.64	0.37
$^{124}Sn \rightarrow {}^{124}Te$	2.62	0.37
$^{128}Te \rightarrow {}^{128}Xe$	2.88	1.32
$^{130}Te \rightarrow {}^{130}Xe$	2.65	0.28
$^{136}Xe \rightarrow {}^{136}Ba$	2.19	0.38

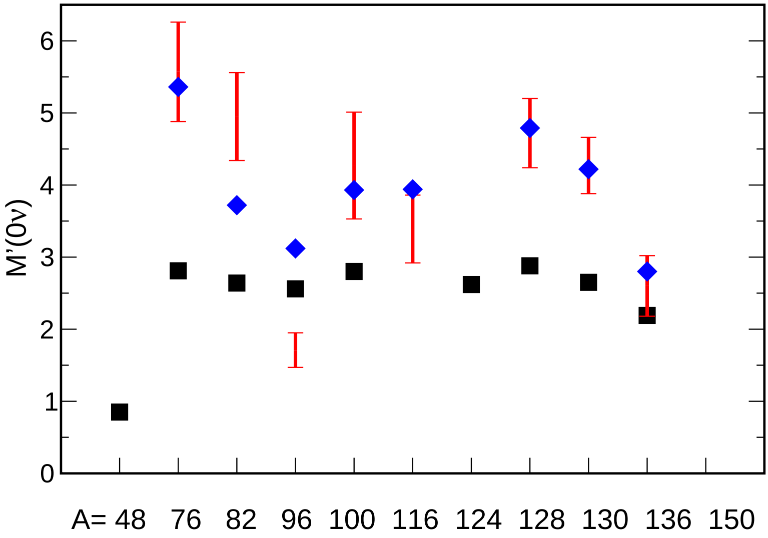
The effective neutrino masses (in eV) corresponds to $T_{\frac{1}{2}} = 10^{25}$ y.
Notice that the Heidelberg-Moscow claim ($T_{\frac{1}{2}} = 2.23^{+0.44}_{-0.31}$ x 10^{25} y) together with our NME leads to an effective neutrino mass of 0.5 eV.

Half-life predictions in the different scenarios for the neutrino mass hierarchy

$T_{\frac{1}{2}}$:	$\langle m_\nu \rangle$=500meV	$\langle m_\nu \rangle$=50meV	$\langle m_\nu \rangle$=5meV
	degenerate	inverted	normal
	10^{25}y	10^{27}y	10^{29}y
$^{48}Ca \rightarrow {}^{48}Ti$	1.6	1.6	1.6
$^{76}Ge \rightarrow {}^{76}Se$	2.1	2.1	2.1
$^{82}Se \rightarrow {}^{82}Kr$	0.7	0.7	0.7
$^{124}Sn \rightarrow {}^{124}Te$	0.7	0.7	0.7
$^{128}Te \rightarrow {}^{128}Xe$	7.0	7.0	7.0
$^{130}Te \rightarrow {}^{130}Xe$	0.3	0.3	0.3
$^{136}Xe \rightarrow {}^{136}Ba$	0.6	0.6	0.6

ISM(black) ; QRPA: Tu(red) , Jy(blue)

UCOM- SRC



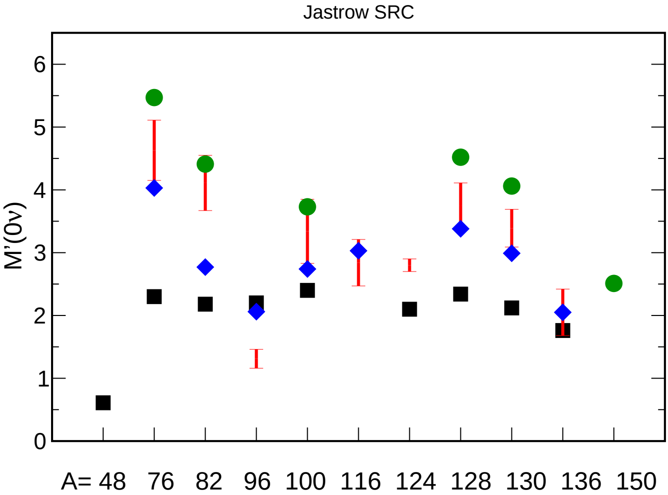
ISM *vs* QRPA NME's

The range of QRPA values shown in the figures derives from the different choices of the valence space and from the use of proper QRPA or (Renormalized)QRPA. The upper tip of the bar and the diamonds correspond to QRPA calculations. g_A=1.25 is adopted in all cases.

The NME's of the two main QRPA groups are now compatible in most decays, which was not the case not so long ago. These are good news.

However, except for ^{96}Zr, ^{124}Sn, and ^{136}Xe, the ISM NME's are systematically smaller than the QRPA ones. Are these bad news?

ISM(black) ; QRPA: Tu(red) , Jy(blue) ; IBM(green)



QRPA results from Rodin, Simkovic, Faessler, and Vogel 07.

QRPA results from Kortelainen and Suhonen 07

IBM; Barea and Iachello 09

ISM; Menendez, Poves, Caurier and Nowacki 09

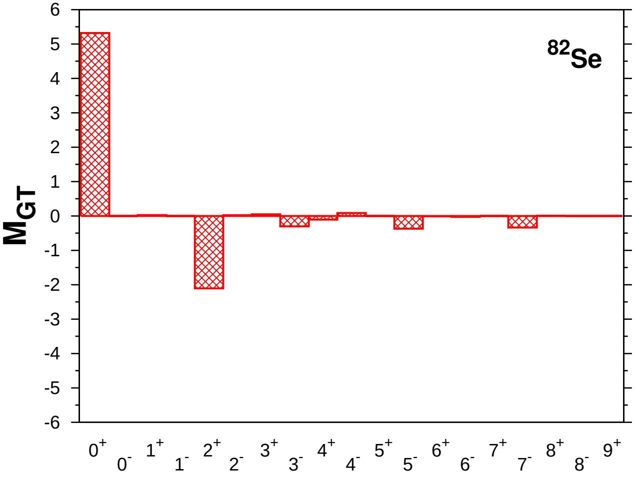
How do the 0ν operators act?

The two body transition operators can be written generically as:

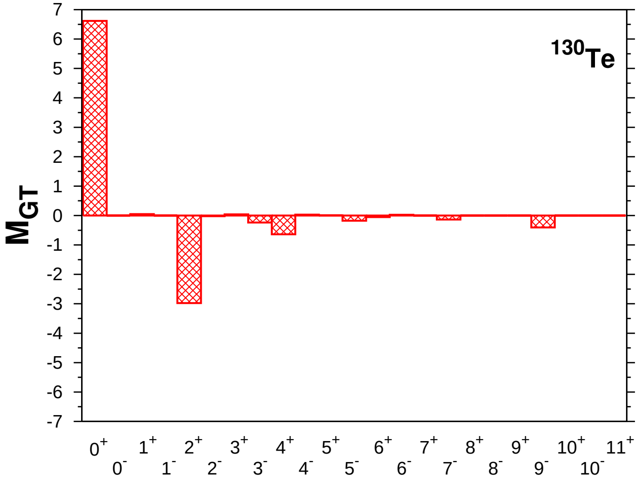
$$\hat{M}^{(0\nu)} = \sum_{J^\pi} \hat{P}^\dagger_{J^\pi} \;\; \hat{P}_{J^\pi}$$

The operators $\hat{P}_{J^\pi}$ annihilate pairs of neutrons coupled to J^π in the parent nucleus and the operators $\hat{P}^\dagger_{J^\pi}$ substitute them by pairs of protons coupled to the same J^π. The overlap of the resulting state with the ground state of the grand daughter nucleus gives the J^π-contribution to the NME. The –a priori complicated– internal structure of these exchanged pairs is dictated by the double beta decay operators.

The contributions to the NME as a function of the J^π of the decaying pair: $^{82}Se \rightarrow {}^{82}Kr$



The contributions to the 0ν matrix element as a function of the **J** of the of the decaying pair : A=130



Pairing Shows Up

These results are very suggestive, because the leading contribution corresponds to the decay of J=0 pairs, whereas the contributions of the pairs with J>0 are either negligible or have opposite sign to the dominant one.

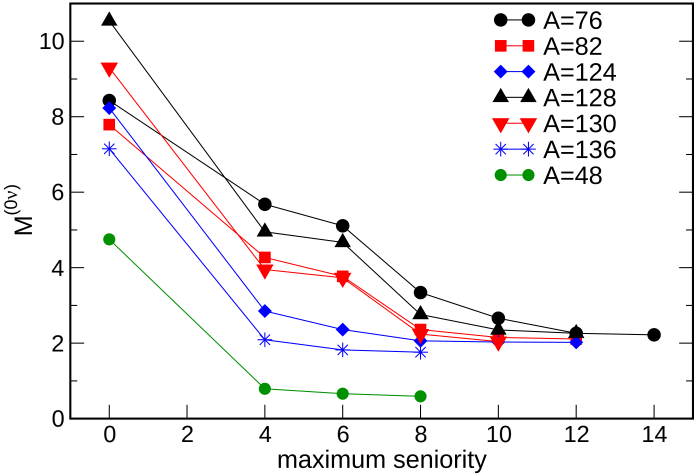
This behavior is common to all the cases that we have studied. It also occurs in the QRPA calculations, in whose context it has been previously discussed by Engel, Vogel et al.

If we went to the limit of pure pairing correlations, i.e. when the initial and final states have generalized seniority zero, there will be no canceling contributions and therefore the matrix element will be maximal.

Pairing Shows Up

Intriguingly, this reveals that the NME's of the neutrinoless double beta decay depend on the pair content of the nuclear wave functions of parent and grand daughter nuclei. If the nuclear wave functions were fully paired, the NME's should become very large. The nuclear correlations of multipole type (mainly quadrupole) may break pairs differently in the initial and final nuclei thus reducing the NME's.

NME's *vs* the maximum seniority content of the ISM WF's



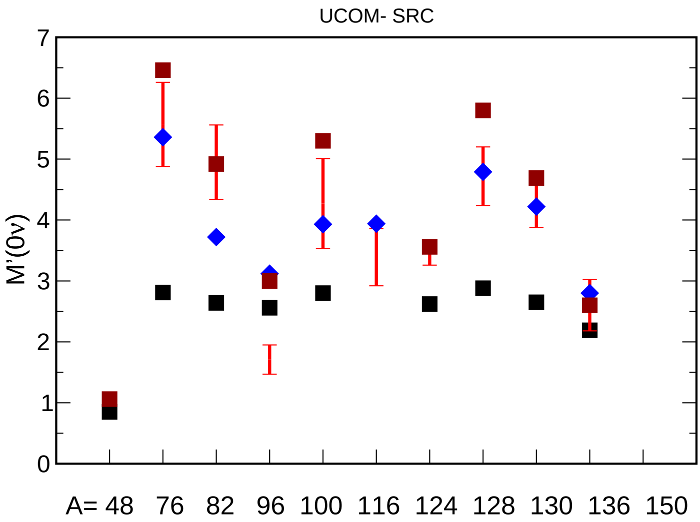
Pay special attention to the s≤4 results because, at leading order, this is the level of ground state correlations present in the QRPA calculations based upon the spherical BCS solution.

The NME's *vs* the maximum seniority of the WF's

Only when the ISM calculations are converged at $s \leq 4$ the QRPA NME's are close to the ISM ones!!

But, in all cases the ISM calculations truncated at $s \leq 4$ and the full QRPA NME's are quite close and what is more telling, have the same global trend

ISM: full(black) , s_m=4(maroon) ; QRPA: Tu(red) , Jy(blue)



ISM *vs* QRPA NME's

- The QRPA results are reasonably close to the ISM ones at s$\leq$4.
- The ISM values at s$\leq$4 are far from converged, except in the A=48, A=96, A=124 and A=136 decays
- Thus, we surmise that, except in these cases, the QRPA overestimates the values of the NME's

ISM *vs* QRPA NME's

- The correlated QRPA ground state can be written as:

$$|QRPA >= N_0 e^S |BCS >$$

where

$$S = \sum_{abcd} C_{abcd} A^{\dagger}_{ab} \overline{A}^{\dagger}_{cd}$$

the A's are the two quasiparticle creation operators.
- The QRPA gs. contains 4n quasiparticle excitations, corresponding to seniority 4n components.
- In order for the QRPA wave functions to make sense, the probability of 4n-quasiparticles (seniority=4n) components must diminish with n, therefore the contributions of high seniority may be quenched in the spherical QRPA description if the quadrupole correlations are strong enough.

The ISM nuclear wave functions in the seniority basis

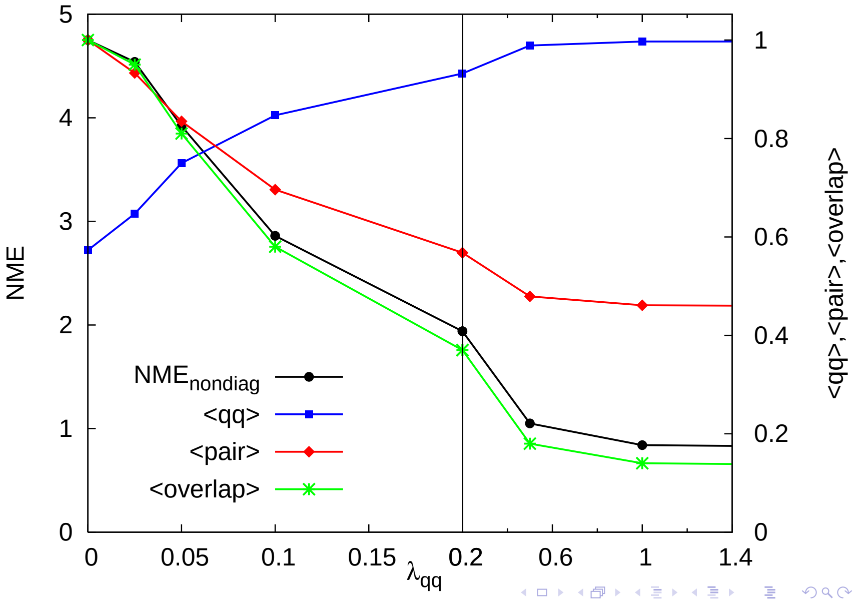
	$s=0$	$s=4$	$s=6$	$s=8$	$s=10$	$s=12$	$s=14$
^{48}Ca	97	3	-	-	-	-	-
^{48}Ti	59	36	4	1	-	-	-
^{76}Ge	43	41	7	8	1	-	-
^{76}Se	26	41	11	16	4	1	-
^{82}Se	50	39	10	1	-	-	-
^{82}Kr	44	41	6	8	1	-	-
^{124}Sn	95	5	-	-	-	-	-
^{124}Te	60	33	6	2	-	-	-
^{128}Te	70	26	3	1	-	-	-
^{128}Xe	37	41	9	10	2	-	-
^{130}Te	79	20	1	-	-	-	-
^{130}Xe	46	39	7	7	1	-	-
^{136}Xe	97	3	-	-	-	-	-
^{136}Ba	72	25	2	1	-	-	-

The role of deformation: The ideal but unreal case of a mirror decay

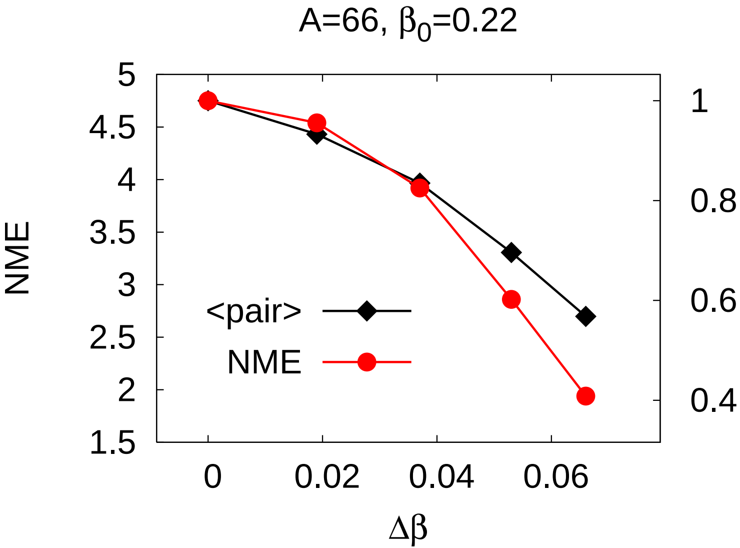
$$^{66}\text{Ge} \rightarrow {}^{66}\text{Se}$$

If we compute both nuclei with the same interaction, they have the same deformation. If we compute the parent with H_0 and the grand daughter with $H_0 + \lambda Q \cdot Q$ we can evaluate the influence of the *differences* of deformation in the NME

$^{66}Ge \rightarrow {}^{66}Se$



$^{66}Ge \rightarrow {}^{66}Se$



Conclusions

- Large scale shell model calculations with high quality effective interactions are available or will be in the immediate future for all but one of the neutrinoless double beta emitters
- We have found that the superfluid correlations in parent and grand daughter favor the neutrinoless decay.
- We have also seen that in the realistic cases, where many other correlations are present, their contributions to the matrix elements come with opposite sign to the the pairing ones.
- In order to take properly into account these cancellations, it is crucial to describe correctly the pair structure of the wave functions.

Conclusions

- State of the art QRPA calculations using the same prescription for the short range correlations are now compatible. The softest possible choice, UCOM, seems to be the more realistic one
- Low seniority truncations s$\leq$4, similar to those present in the spherical QRPA approaches based in a BCS treatment of the pairing interaction, are shown to fall short in the capture of the proper correlations, and hence to overestimate the nuclear matrix elements in several decays.
- The difference in the amount of quadrupole correlations in the ground states of parent and grand daughter hinders the transition.